Abstract

In this thesis we present several new algorithms for dynamic graph problems. The common theme of the problems we consider is connectivity. In particular, we study the maintenance of connected components in a dynamic graph, and the Steiner tree problem over a dynamic set of terminals.

First, we present an algorithm for decremental connectivity in planar graphs. It processes any sequence of edge deletions intermixed with a set of connectivity queries. Each connectivity query asks whether two given vertices belong to the same connected component. The running time of this algorithm is optimal, that is, it handles any sequence of updates in linear time, and answers queries in constant time. This improves over the best previously known algorithm, whose total update time is $O(n \log n)$.

Then, we study the dynamic Steiner tree problem. In this problem, given a weighted graph $G$ on a vertex set $V$ and a dynamic set $S \subseteq V$ of terminals, subject to insertions and deletions, the goal is to maintain a constant-approximate Steiner tree spanning $S$ in $G$. For general graphs and every integer $k \geq 2$, we show an $(8k - 4)$-approximate algorithm, which processes updates in $O(k n^{1/k} \log^4 n)$ amortized expected time. In the case of planar graphs we show a different solution, whose amortized update time is $O(\varepsilon^{-1} \log^6 n)$ and the approximation ratio is $4 + \varepsilon$.

Finally, we study graph connectivity in a semi-offline model. We consider a problem, in which the input is a sequence of graphs $G_1, \ldots, G_t$, such that $G_{i+1}$ is obtained from $G_i$ by adding or removing a single edge. In the beginning, this sequence is given to the algorithm for preprocessing. After that, the algorithm should efficiently answer queries of one of two kinds. A forall$(a, b, u, w)$ query, where $1 \leq a \leq b \leq t$ and $u, w$ are vertices, asks whether $u$ and $w$ are connected with a path in each of $G_a, G_b, \ldots, G_b$. Similarly, an exists$(a, b, u, w)$ query asks if the given vertices are connected in any of $G_a, G_b, \ldots, G_b$. For forall queries, we show an algorithm that after preprocessing in $O(t \log t (\log n + \log \log t))$ expected time answers queries in $O(\log n \log \log t)$ time. In the case of exists queries, the preprocessing time is $O(m + nt)$ and the query time is constant.

Key words: dynamic graph algorithms, dynamic connectivity, decremental connectivity, Steiner tree

AMS Classification: 05C85 Graph algorithms, 68P05 Data structures, 68Q25 Analysis of algorithms and problem complexity, 68W40 Analysis of algorithms