Abstract of the PhD thesis

Strong Interaction Corrections to the Weak Radiative B-Meson Decay at Order $\mathcal{O}(\alpha_s^2)$ with Exact Dependence on the $c$-Quark Mass

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The process $\bar{B} \to X_s \gamma$ is known to provide important constraints on extensions of the Standard Model (SM). The present SM prediction for its CP- and isospin-averaged branching ratio reads $\mathcal{B}(\bar{B} \to X_s \gamma)^{\text{SM}} = (3.36 \pm 0.23) \cdot 10^{-4}$. It agrees very well with the current experimental average $\mathcal{B}(\bar{B} \to X_s \gamma)^{\text{exp}} = (3.43 \pm 0.22) \cdot 10^{-4}$. The experimental accuracy is expected to improve in a significant manner after the Belle-II experiment begins collecting data within the next few years. Consequently, theoretical calculations must also be upgraded to match the experimental precision.

A considerable contribution to the current theoretical uncertainty originates from the fact that some of the Next-to-Next-to-Leading-Order strong interaction corrections (called $K_{17}^{(2)}$ and $K_{27}^{(2)}$) have not yet been calculated for an arbitrary value of the charm and bottom quark mass ratio $m_c/m_b$. Instead, known results for these corrections at $m_c = 0$ and for $m_c \gg m_b/2$ serve as a basis for an interpolation in $m_c$, which introduces around $\pm 3\%$ uncertainty into $\mathcal{B}(\bar{B} \to X_s \gamma)^{\text{SM}}$.

In order to remove this uncertainty, determining the exact dependence of $K_{17}^{(2)}$ and $K_{27}^{(2)}$ on the $c$-quark mass is necessary. In the language of Feynman diagrams with unitarity cuts, four-loop diagrams with two mass scales ($m_c$ and $m_b$) need to be evaluated. The necessary ultraviolet counterterms involve three-loop two-mass-scale diagrams that must be calculated up to $\mathcal{O}(\varepsilon)$ in the dimensional regularization parameter $\varepsilon$.

In the present thesis, we evaluate [1] the exact dependence on the $c$-quark mass of all the necessary ultraviolet-counterterm diagrams that contribute to the yet-unknown parts of $K_{17}^{(2)}$ and $K_{27}^{(2)}$. These corrections originate from interferences of four-quark and photonic dipole operators. They are currently responsible for the main uncertainty in the perturbative contribution to $\mathcal{B}(\bar{B} \to X_s \gamma)^{\text{SM}}$.

Apart from the calculation for arbitrary $m_c$, we also evaluate many of the necessary counterterm contributions at $m_c = 0$, and present them to all orders in $\varepsilon$ wherever possible. Our results have contributed to the evaluation of the $m_c = 0$ boundary for the interpolation, and thus to the recently published updated phenomenological analysis of $\mathcal{B}(\bar{B} \to X_s \gamma)^{\text{SM}}$ [2].

The thesis contains many technical details that have not been presented elsewhere, namely explicit expressions for all the relevant quantities in terms
of the master integrals, as well as results for these integrals obtained using several different methods, involving Mellin-Barnes techniques and differential equations.
