ASTROMETRY OF
THE OGLE-III DATA

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Ph.D. Thesis

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Section 3 presents slightly extended results already published in the papers by Poleski et al. (2011) and Poleski et al. (2012).

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Abstract

The thesis presents the astrometric analysis of the data collected during the third phase of the Optical Gravitational Lensing Experiment. The area of 54 square degrees towards the Magellanic Clouds was analyzed. The clean and complete sample of the stars with proper motions higher than 0.1 arcsecond per year was prepared and analyzed. A catalog of over 6.2 million stellar proper motions is presented and discussed. For over 110 000 stars also the parallaxes are presented. Separate analysis was performed in four fields towards the Galactic bulge, in which a double red clump is observed. The double red clump is caused by an X-shaped structure. The proper motions of the stars in the two arms of this structure were compared. Significant differences in mean longitudinal proper motions were found. The dispersions of the proper motions in both arms of the structure were derived for the first time.

Keywords: astrometry — catalogs — galaxy: bulge — galaxy: kinematics and dynamics — globular clusters: 47 Tuc — Magellanic Clouds — parallaxes — proper motions
Summary in Polish

Astrometria danych OGLE-III


Celem niniejszej pracy było wyznaczenie ruchów własnych i paralaks gwiazd obserwowanych w ramach trzeciej fazy projektu OGLE (ang. Optical Gravitational Lensing Experiment – Optyczny Eksperyment Soczewkowania Grawitacyjnego). Obserwacje OGLE-III prowadzone były przy użyciu Teleskopu Warszawskiego o średnicy 1,3 m, który znajduje się w Obserwatorium Las Campanas (Chile). Teleskop wyposażony był w ośmiodetektorową kamerę mozaikową, a obserwacje prowadzone przez osiem lat. Niniejsza analiza obejmuje pola w Obłokach Magellana (łącznie 54 stopnie kwadratowe) oraz wybrane pola obserwowane w kierunku zgrubienia centralnego Galaktyki (1,3 stopnia kwadratowego).

Parametry astrometryczne gwiazd obserwowanych w kierunku Obłoków Magellana zostały wyznaczone na podstawie wcześniej zredurowanych obrazów nieba z projektu OGLE-III. Pierwszym celem przedstawionej pracy było wykrycie oraz określenie parametrów fizycznych gwiazd o największych ruchach własnych. Ze względu na stosunkowo niewielką liczbę takich gwiazd była możliwa szczegółowa weryfikacja każdego obiektu.

Dla gwiazd z pół w Obłokach Magellana, poza streszczonej wyżej analizą gwiazd o największych ruchach własnych, sporządzono i przeanalizowano katalog ruchów własnych ponad sześciu milionów gwiazd. Na podstawie diagramu Hertzsprunga-Russella wyselekcjonowano ponad 200 dalszych białych karłów. Parametry fizyczne zbadane zostały także na podstawie diagramu zredukowanych ruchów własnych. Diagram ten pozwala oddzielić gwiazdy ciągu głównego należące do dysku galaktycznego od tych należących do populacji hało.

Dzięki dużej liczbie gwiazd w zaprezentowanym katalogu ruchów własnych możliwe było wyszukiwanie par gwiazd o wspólnym ruchu własnym. Przeprowadzone zostały badania statystyczne, dzięki którym określono, które spośród par gwiazd o podobnym ruchu własnym są losową koincydencją, a które składają się z fizycznie związanych ze sobą gwiazd.

pływowym 47 Tuc. Wśród wykrytych gwiazd zmiennych są trzy, które nie były wcześniej znane i mają ruch własne taki jak gromada. Pierwsza z nich to dwumodalna gwiazda zmienna typu SX Phe. Być może obserwowane u niej mody są pulsacjami w czwartym i piątym radialnym modzie harmonicznym. Druga to gwiazda zmienna typu SX Phe, która pokazuje jeden mod pulsacji. Trzecia to gwiazda należąca do słabo zbadanego typu czerwonych maruderek wykazująca zmienność typową dla rotujących gwiazd zaplamionych. Czerwone marudery to gwiazdy, które są czerwieniące od gałęzi olbrzymów gromady i mają jasności zbliżone do podolbrzymów.


Oddzielnie omówione zostały ruchy własne gwiazd zmiennych obserwowanych w kierunku Obłoków Magellana. Dość nieoczekiwanie okazało się, że zaprezentowany katalog umożliwił określenie natury pewnego typu niebieskich gwiazd zmiennych. W próbce takich gwiazd znalezionych w danych z drugiej fazy projektu OGLE wykryta została grupa gwiazd nieznacznie czerwieniutkich niż pozostałe obiekty. Na podstawie niniejszego katalogu stwierdzono, że nie są to gwiazdy zmienne z Wielkiego Obłoku Magellana, lecz pobliskie gwiazdy z Galaktyki. Mierzone dla tych gwiazd zmiany jasności nie były realnym efektem, ale artefaktem powodowanym przez użytą metodę fotometryczną. Katalog ruchów własnych wykorzystany został do wyselekcjonowania gwiazd z Wielkiego Obłoku Magellana, które znajdują się w klasycznym pasie niestabilności, ale nie wykazują pulsacji.

Drugim zagadnieniem podjętym w pracy jest pomiar ruchów własnych gwiazd w kierunku zgrubienia centralnego Galaktyki. Diagramy barwa-jasność wykonane w niektórych polach pokazują obecność dwóch zagęszczeń czerwonych olbrzymów (amg. red clump). Jest to dowód na istnienie w zgrubieniu centralnym struktury o kształcie X. Wybrane zostały cztery pola, w których ta struktura jest widoczna, a jednocześnie duża liczba obserwacji OGLE-III w tych polach umożliwia dokładne wyznaczenie ruchów własnych gwiazd. Obrazy nieba zostały zredukowane programem napisanym przez autora pracy,
umożliwiającym wyznaczenie pozycji gwiazd. Na podstawie zmierzonych ruchów własnych zbadane zostały różnice statystyczne w kinematyce dwóch ramion struktury o kształcie X. Po raz pierwszy wykryte zostały różnice w dyspersjach ruchów własnych: dyspersje w bliższym ramieniu są większe niż w dalszym ramieniu. Wyraźne różnice widoczne są również w średnich ruchach własnych w kierunku równoległym do dysku Galaktyki. W kierunku prostopadłym różnice te są w granicy błędów zgodne z zerem.
Chapter 1

Introduction

The goal of astrometry is to precisely measure the positions and movements of celestial bodies. For a single star the two motions that carry scientific information are parallax and proper motion. The positions of stars as observed from the ground are also affected by atmospheric refraction, aberration, precession, and nutation of the Earth’s axis. In the following sections we describe how we removed the impact of these effects on measured positions.

The effect of parallax is seen when an apparent position of a star changes as the Earth travels around the Sun. The value of parallax angle is equal to the change of position caused by moving an observer by 1 AU (1 AU = 149.6 \cdot 10^6 \text{ km}) away from the Sun. The parallax is measured in arcseconds and its reciprocal is the distance to the star expressed in parsecs (one parsec is the number of arcseconds in a radian multiplied by 1 AU). The observed change of stellar position due to parallax has a shape which depends on position on the sky. For a star located exactly at one of the ecliptic poles, the parallax motion draws a circle in the sky (assuming that the Earth’s orbit is a circle). For stars on the ecliptic the parallax draws a line segment. For intermediate ecliptic latitudes, the parallax traces a more or less flatten ellipse. We note that the South Ecliptic Pole coincides with the Large Magellanic Cloud (LMC) on the sky, while the Galactic bulge is close to the ecliptic.

The proper motion is an angular velocity of a star. In the equatorial coordinates—right ascension (\( \alpha \)) and declination (\( \delta \))—it has two components; one parallel to the celestial equator (\( \mu_\alpha \)) and the other which is perpendicular (\( \mu_\delta \)). Because the angular scale changes with declination (\( \delta \)), one typically provides \( \mu_\alpha \cos \delta \) and \( \mu_\delta \). For convenience we denote
the first of them as \( \mu_{\alpha*} \). Thus, the total proper motion is defined as \( \mu = \sqrt{\mu_{\alpha*}^2 + \mu_{\delta}^2} \). If the parallax is known, the total proper motion can be changed into transverse velocity of a star, according to formula \( v_t = 4.74 \cdot \mu/\pi \), where \( v_t \) is in km/s, \( \mu \) in arcsec/yr, and \( \pi \) is in arcsec.

The apparent position of a star in equatorial coordinates is affected by the proper motion, parallax, and differential refraction. It is given by the following formulae (e.g., Kovalevsky & Seidelmann, 2004, Sec. 6):¹

\[
\alpha = \alpha_0 + \mu_\alpha t + \frac{r \sin p \tan z + \pi \sin \gamma \sin \beta}{\cos \delta} \quad (1.1)
\]

\[
\delta = \delta_0 + \mu_\delta t + r \cos p \tan z + \pi \sin \gamma \cos \beta \quad (1.2)
\]

where \( t \) is time of observation, \( \alpha_0 \) and \( \delta_0 \) are equatorial coordinates for \( t = 0 \) which corresponds to epoch J2000.0, \( r \) is differential refraction coefficient, \( p \) is the angle between the direction of the parallax shift and the direction to the North Celestial Pole, \( z \) is the zenith distance, \( \gamma \) is the angular distance to the Sun, and \( \beta \) is the angle between the direction of the parallax shift and the direction to the North Celestial Pole. The refraction, aberration, precession, and nutation affect each star in the image equally and thus were removed by the grid fitting procedures used. In principle, the values of \( p, z, \gamma, \) and \( \beta \) may be different in different parts of the CCD chip and were calculated separately for every epoch and every star using van Flandern & Pulkkinen (1979) ephemerides. For distant stars (\( \pi > 1 \) kpc), the effect of parallax is negligible for current observing capabilities. If one tries to find it for such stars or when not many epochs for a given star were collected, it is possible to overfit which may result in erroneous results. Because of that, we often used the above equations neglecting the parallax effect:

\[
\alpha = \alpha_0 + \mu_\alpha t + \frac{r \sin p \tan z}{\cos \delta} \quad (1.3)
\]

\[
\delta = \delta_0 + \mu_\delta t + r \cos p \tan z \quad (1.4)
\]

To find the parallax, one needs very accurate observations separated by roughly half of a year. Measurements of proper motions are easier if the stellar positions from different

¹Soszyński et al. (2002) and Sumi et al. (2004) presented these equations with errors, notwithstanding their results were correct.
epochs are available. The longer the time difference between observations the better is the accuracy of derived proper motion, thus before the CCD era astronomers used epochs separated typically by a few dozen years. Decades ago the proper motions and parallaxes were expressed in arcsec/yr and arcsec, respectively. The pixel scales of currently used astronomical cameras are fractions of 1 arcsec and the stellar centroids are measured with accuracy around a hundred times better. It turned out that milliarcseconds (abbreviated to mas) are more convenient units than arcseconds.

The star with the highest known parallax is Proxima Centauri ($\pi = 769$ mas, $\mu = 3.853$ mas/yr) and it is a part of a triple system. The highest proper motion star is Barnard’s Star ($\pi = 546$ mas, $\mu = 10.358$ mas/yr), which is also the closest star except the Proxima Centauri system.

1.1 Historical overview

As stated above, the proper motions are much easier to be measured than parallaxes. The first measurements of proper motions critically depended on a time baseline of observations.

Hipparchos of Nicaea was the first who noticed that the position of the North Celestial Pole changes with respect to the “fixed” stars. His discovery, done in 150 BC, was possible because the observations of Spica were recorded 160 yr earlier. Hipparchos also created a catalog containing positions of 1080 stars. Claudius Ptolemy developed mathematical descriptions of movements of celestial bodies, including the sphere of the “fixed” stars.

The star catalogs were also prepared by Al-Sufi (960 AD) and Ulugh Beg (1430). The improvement of the measuring technique was devised by Tycho Brahe in XVI century. Thanks to the scales on sextant- and quadrant-type instruments his observations had an accuracy in the range 15–35$''$ and were a few tens times more accurate than the earlier observations.

In 1718 Edmund Halley showed that Arcturus, Sirius, and Aldebaran changed their positions since Ptolemy published his catalog (1600 years earlier). Recently measured proper motions of these stars are 199.3 mas/yr, 1339.4 mas/yr, and 2279.4 mas/yr, respectively (van Leeuwen, 2007).

The next improvement in astrometric precision was gained by using the telescopes. The first star catalog based on telescopic observations was prepared by John Flamsteed in
1725.

The ultimate goal of all astrometric measurements done in XVIII century was to measure the suspected effect of parallax. In 1728 James Bradley, trying to measure the parallax, discovered the aberration displacement. Twenty years later the same astronomer found nutation. Further measurements of the proper motion for different stars allowed William Herschel to determine the solar apex direction which points to the constellation of Hercules. We note that the direction of the solar apex is nowadays established with an accuracy of a few degrees (Francis & Anderson, 2009; Schönrich et al., 2010).

The usage of the telescopes with long-focus and the heliometers led to the first measurements of parallax which were independently performed by Friedrich Wilhelm Bessel, Friedrich Georg Wilhelm von Struve, and Thomas Henderson in 1838. They measured the parallax values for 61 Cyg ($\pi = 286$ mas), Vega ($\pi = 130$ mas), and $\alpha$ Cen ($\pi = 769$ mas), respectively.

During the XIX century, the main instruments used for astrometric measurements were transit circles. They allowed precise position determination for only one star at a time. In the middle of XIX century, the photography became standard technique in astronomy. Especially if Schmidt telescopes were used, many stars could be measured at the same time, which significantly improved observing capabilities.

The breakthrough in astrometric precision was achieved by the space mission Hipparcos launched by the European Space Agency in 1989. The satellite was equipped with a Schmidt telescope which main mirror was divided into two halves. The distance between the parts of the sky observed by each half was 58°. This, combined with the rotation of the satellite, allowed comparison of the position of a given star to positions of different stars lying 58° away and studying the whole sky. The telescope was observing till 1993. Four years later two astrometric catalogs were presented (Perryman & ESA, 1997). They were named after historical astronomers: Hipparcos and Tycho Catalogs. The catalogs turned out to contain systematic errors. Ten years later van Leeuwen (2007) presented a new analysis of the raw original data. The main difference between these two reductions was the treatment of small discontinuities in the satellite motion. It allowed van Leeuwen (2007) to reduce the systematic errors. The accuracy of parallaxes of the new reduction is around 1 mas for stars as bright as $H_p = 9$ mag and reaches 0.1–0.2 mas for the brightest stars. They also showed that the errorbars provided were only of statistical origin and no
systematic trends were seen.

The Hubble Space Telescope (HST) was launched one year after the Hipparcos satellite. It is equipped with a 2.4 m mirror with a number of different instruments, including imaging cameras. Thanks to five servicing missions, the telescope possess state-of-the-art instruments. Their fields of view are very small compared to the ground-based cameras, e.g., Wide Field Channel of Advanced Camera for Surveys gives $202'' \times 202''$ with a pixel scale of $0''050$. Thanks to the long mission and multiple pointings in the same part of the sky, very accurate measurements could be obtained. The accuracy of the HST measurements increased since the work by Anderson & King (2000) who showed how important is the modelling of the point spread function (PSF). In many cases the proper motions based on the HST observations are derived from a few epochs only with many dithering observations in each epoch. In the present thesis, we will compare the HST astrometry for the LMC, 47 Tucanae (NGC104) globular cluster and the vicinity of the Baade window with our measurements. The most impressive HST proper motion measurement was that of M31 galaxy which had an accuracy of around $0.02\text{ mas/yr}$ (Sohn et al., 2012).

The successor of the Hipparcos mission is planned to be launched in 2013. The Gaia mission goals at measuring positions of $V = 10$ mag stars with a remarkable accuracy of a few microarcseconds. For a faint limit of the mission ($V \approx 20$ mag), the planned accuracy is a few parts of mas.

### 1.2 Microlensing surveys and proper motions

Paczyński (1986) suggested that the intensive and long-term photometric survey could verify one of the hypothesis explaining the dark matter phenomenon. If the Galactic dark matter was composed of unseen objects with masses between $10^{-6}M_\odot$ and $10^2M_\odot$, then observer monitoring an order of a few millions stars should observe gravitational microlensing events with timescale between two hours and a few hundred days. At the time the main problem with implementation of this idea was finding CCD camera efficient enough and reducing the collected data at the rate comparable to their acquisition.

In the beginning of 1990s, three independent groups started their photometric surveys: EROS (Expérience de Recherche d’Objets Sombres; Aubourg et al., 1993), MACHO (MAssive Compact Halo Objects; Alcock et al., 1992), and OGLE (Optical Gravitational
Lensing Experiment; Udalski et al., 1992). The first two of them were monitoring the fields in the LMC as originally suggested by Paczyński (1986). The OGLE survey used different strategy. At this time no microlensing events were known, thus the negative result of the LMC monitoring could be interpreted as either falsification of the hypothesis describing the dark matter or inability to identify microlensing events. Paczyński (1991) were aware of that problem and proposed observations of the Galactic bulge as a sky area where high density of background stars should result in microlensing by foreground disk objects no matter what the composition of the dark matter is. The OGLE project lead by Prof. Andrzej Udalski followed that prescription.

All three microlensing surveys announced discoveries of potential microlensing events in 1993 (Aubourg et al., 1993; Alcock et al., 1993; Udalski et al., 1993). Two candidate microlensing events announced by EROS towards the LMC turned out to be variable stars (Ansari et al., 1995; Beaulieu et al., 1995). This fact did not stop the observational efforts towards discovering more microlensing events.

The final analysis of the MACHO project gave the fraction of dark matter in massive compact objects of 20% (Bennett, 2005). The claimed number of microlensing events observed towards the LMC was around ten. The EROS group analyzed both the LMC and Small Magellanic Cloud (SMC) (Tisserand et al., 2007). Contrary to MACHO, they reported only one microlensing event and found the upper limit of the fraction of dark matter in massive compact objects of 8%.

The second and third phases of the OGLE survey except the bulge observed also the LMC and SMC. The microlensing events observed toward the LMC and SMC were presented in a series of papers (Wyrzykowski et al., 2009, 2010, 2011a,b). In total, six events in the LMC and four events in the SMC were found. The reported upper limit of the fraction of dark matter in massive compact objects was 4% (Wyrzykowski et al., 2011b), what ruled out the hypothesis that such objects significantly contributed to the dark matter.

Testing the dark matter content was not the only area of research in which microlensing surveys data were used. The most profiting one was variable stars research which was suggested as a by-product by Paczyński (1986). The summary of different research topics investigated with the microlensing surveys data can be found in Paczyński (1997), Udalski (2009), and Mao (2012). One of the research areas in which microlensing survey data were used was measuring of the proper motions and parallaxes for a large number of stars. The
first paper on stellar proper motions based on EROS data (EROS Collaboration et al., 1999) relied on a two-epoch sub-project conducted in 413 square degrees. The images were collected when the standard EROS observations could not be performed because of the main target visibility conditions. The result of this sub-project was a discovery of two L-type dwarfs.

Eyer & Woźniak (2001) were first who noticed that artifacts produced by the Difference Image Analysis (DIA) method may be used to search for the high proper motion (HPM) stars. In their preliminary search, they investigated one of the OGLE-II bulge fields. Alcock et al. (2001) used MACHO data to search for HPM stars in the direction of the Galactic bulge and Magellanic Clouds. Using Eyer & Woźniak (2001) method, they found altogether 154 new HPM stars (only one of them was toward the SMC). Soszyński et al. (2002) conducted a similar search in the OGLE-II data for the Magellanic Clouds. Altogether 3 053 stars had proper motions measured down to 4 mas/yr. For 38 stars parallaxes with accuracy better than 5σ were measured. In the thesis we investigate the HPM stars in the OGLE-III Magellanic Clouds fields. In these fields we also wanted to present the catalog of all stars for which proper motions could be reliably measured and show astrophysical contexts in which such a catalog can be used.

The next paper by the EROS group (Goldman et al., 2002), similarly to the previous one, used images which were not taken during the normal survey observations. They searched for halo white dwarfs (WDs) but no such object was found. They estimated contribution of WDs to halo mass to be below 5%, what further strengthened the results of Wyrzykowski et al. (2011b).

In their preliminary study, Sumi et al. (2003) divided the red clump (RC) stars in the Baade’s window into bright (i.e., located closer) and faint (i.e., located further) subsamples. The absolute brightness of RC stars is almost constant and their abundance is high, thus, they serve as good standard candles. Sumi et al. (2003) used the proper motions of these subsamples in one of the bulge OGLE-II fields to find the evidence for the Galactic bar rotation. Significant difference of the mean proper motion between the subsamples was found, what was in a very good agreement with model predictions by Mao & Paczyński (2002) who assumed bar tangential streaming motion of 100 km/s. We note that there are no quasars known behind the bulge, thus all the proper motion measurements in bulge fields, except the ones based on Hipparcos satellite data, are relative not absolute.
The very important proper motion study was conducted by Sumi et al. (2004). They based their measurement on the OGLE-II images taken in the bulge fields. Their catalog contained the proper motions for more than 5 million stars within the $I$-band magnitude range of 11–18 and located in 11 square degrees observed by the OGLE-II. Sumi et al. (2004), except presenting the catalog of proper motions, showed also the dispersions of the proper motions calculated based on this catalog. Previously the only other useful determinations on this parameters were based on HST observations (e.g., Kuijken & Rich, 2002) and had better accuracy but observed much smaller fields. Rattenbury & Mao (2008) cross-matched the highest proper motions stars from catalogs by Alcock et al. (2001) and Sumi et al. (2004) with the infrared surveys in order to find photometric distances and luminosity classes of these stars. Rattenbury et al. (2007) compared particle simulation of the Galactic bulge with the proper motion dispersions calculated using Sumi et al. (2004) catalog. They found rough agreement between predicted and observed values. Similarly to Rattenbury et al. (2007), we planned to measure the proper motion statistics separately for two arms of the X-shaped structure.

Possibly very fruitful application of time resolved observations in dense stellar fields was proposed by Paczyński (1995, 1998). If a nearby HPM star microlenses the more distant one (the source), and high spatial resolution is achieved, one could observe both the brightness changes as well as changes of the stellar centroid. The change of the centroid position depends on the lens mass and if such an event is well observed, the mass of the lens can be measured. This is the only one known direct method of measuring masses of stars which are not members of binary systems. The pair of stars which would be aligned and the approximate moment when it occurs may be found in advance. More detailed considerations were performed by Gould (2000) and the first predictions were calculated by Salim & Gould (2000). Recently Proft et al. (2011) searched for such pairs. The proper motions of lensing stars were taken from, among others, OGLE-II bulge catalog of Sumi et al. (2004). However, the positions of sources were taken from the PPMXL catalog. The fact that Proft et al. (2011) used one catalog for source position and different catalogs for lensing objects proper motions was a major disadvantage of their investigation. This hampers the searches of star alignment because of the plausible coordinate differences between the catalogs. The minimum dispersion of the coordinate differences between two catalogs with high precision astrometry—Two Micron All Sky Survey (2MASS Skrutskie
et al., 2006) and The third U.S. Naval Observatory CCD Astrograph Catalog (UCAC3)—is 70 mas (Zacharias et al., 2010). In most cases this dispersion is around 100 times larger than the typical radius of the Einstein ring\(^2\).

1.3 The X-shaped structure in Galactic bulge

The bulge of the Milky Way is only such a structure in which stars can be well resolved and studied in detail. Such studies allow characterization of the bulge structure, kinematics, chemical composition, age, and formation history.

There are multiple lines of evidence that the bulge contains a boxy bar with a near end in the first Galactic quadrant \((0^\circ < l < 90^\circ)\). This can be seen as a change of the dereddened brightness of the RC giants with longitude (Stanek et al., 1994). The near- and far-infrared sky brightness shows a perspective effect of the bar (Dwek et al., 1995). The galactic model, which takes into account the bar, better explains the observed microlensing depth than the model without the bar (Kirága & Paczyński, 1994). Also the streaming proper motions of the RC stars, as presented by Sumi et al. (2003), are best explained by the bar model. Recently, Howard et al. (2008, 2009) used the radial velocities of the bulge stars observed at \(b = -4^\circ\) and \(b = -8^\circ\) to demonstrate that the bar rotation is cylindrical, what is characteristic for the boxy bars.

In 2010, two papers were published showing an evidence for the existence of two separate RC structures in the color-magnitude diagrams (CMDs) at \(|b| > 5^\circ\). Nataf et al. (2010) analyzed optical CMDs from the OGLE-III survey. McWilliam & Zoccali (2010) used both the infrared photometry (2MASS survey and data from 3.6 m NTT telescope) and optical photometry (OGLE-II survey and data from 2.2 m MPG/ESO telescope) to search for the double RC. They also discussed the possibility that the two RCs could differ in age or chemical composition. The results of both investigations were consistent—in the fields with \(|b| > 5^\circ\) the RC is split into two structures, which have the same colors but brightness different by \(\approx 0.5\) mag. The best explanation found was that the bar has an X-shaped structure with one arm closer to the Sun (brighter RC) and the second one further away (fainter RC). Due to the extent of each arm along the line of sight direction and the arms merge in fields close to the Galactic plane. The line of sight separation of the arms is

\(^{2}\)Einstein ring radius defines an angular distance between the source and the lens in microlensing event. It equals 1 mas for 0.5 \(M_\odot\) star which is 4.3 kpc away and microlenses the source at a distance of 8.6 kpc.
≈ 1.5 kpc at $|b| = 5^\circ$, and increases as we look further away from the Galactic plane. We note that well known region of low extinction called the Baade Window ($l = 1.0^\circ$, $b = -3.9^\circ$) does not show clear signature of two RCs, which is the main reason why the X-shaped structure was not known before. The most detailed view of the X-shaped structure was presented by Saito et al. (2011), who used the 2MASS data to construct the density maps. The images of two example edge-on galaxies, which show the X-shaped structures are presented in Fig. 1.1. Our view of the Galactic X-shaped structure is probably comparable to the view of the observer who would be located in coordinates ($-65^\circ$, $0^\circ$) in NG C3390.

McWilliam & Zoccali (2010) not only studied the CMDs but also analyzed the proper motions published by Vieira et al. (2007). Using photographic plates spanning 21 years, Vieira et al. (2007) calculated the proper motions of the stars in the Plaut field ($l = 0^\circ$, $b = -8^\circ$). Based on 326 stars from the brighter RC and 365 from the fainter one, McWilliam & Zoccali (2010) found a difference in proper motions of $0.19 \pm 0.19 \text{ mas/yr}$ in longitudinal direction and $0.51 \pm 0.18 \text{ mas/yr}$ in latitudinal direction.

Our goal was to derive proper motions of stars belonging to the two arms of the X-shaped structure in the Galaxy. We calculated the differences in proper motions of the two arms and compared their proper motion dispersions. These quantities can be used to constrain the models of Galactic bulge gravitational potential.
Figure 1.1: Examples of X-shaped structures in two edge-on galaxies. Top panel shows NGC3390 while the bottom one presents NGC4469. In each part the upper panel shows the optical image from the DSS survey, the middle one—the infrared image in the $K_s$ band, and the bottom one—the $K_s$-band image with smooth profile removed. The contours in the middle panel are spaced by 0.5 mag/arcsec$^2$ with the surface brightness of the faintness profile indicated in the lower right. Images taken from Bureau et al. (2006).
Chapter 2

Observations

2.1 Observing setup and strategy

The data analyzed in this thesis were collected during the third phase of the OGLE project. This phase lasted since June 2001 until May 2009. All the data were collected using the 1.3-m Warsaw Telescope (f/9.2) located at Las Campanas Observatory, Chile. The telescope control system was described by Udalski et al. (1997). The fact that the twin 6.5-m Magellan Telescopes are located at the same observatory and that construction of the 24.5-m Giant Magellan Telescope has already begun shows that Las Campanas Observatory is the superior site for astronomical research (e.g., Bakos et al., 2012). The latitude of the observatory restricts the minimum airmasses at which the LMC and SMC are observed to 1.26 and 1.34, respectively. The bulge fields are crossing the meridian close to the zenith, thus minimum airmass for those fields is 1.00.

During the OGLE-III survey the telescope was equipped with a “second generation” camera. The camera consisted of eight STe ST-002a CCD chips with 2048 × 4096 pixels. The full camera size was 8192 × 8192. The pixel size was 15µm which resulted in 0.26 arcsec/pix scale and 35′ × 35′ total field of view.

The observations were conducted in the V- and I-band filters, which closely resemble the standard ones. The quantum efficiency of the observing setup is higher when the I-band filter is used, thus, most of the observations were taken using this filter. In the bulge fields 98.4% of observations were taken in the I-band. The numbers for the LMC and SMC are 85.7% and 89.2%, respectively. The total observed sky area was 39.8 square degrees for the LMC fields and 14.2 square degrees for the SMC fields. For the LMC fields between 385 and
637 epochs with $I$ filter were taken. All the SMC fields except two were imaged between 619 and 762 times in the $I$-band. For the field SMC128 as many as 1228 epochs were secured. This field was more frequently observed during 2005 (513 epochs) and 2006 (181 epochs), when the microlensing event OGLE-2005-SMC-001 occurred. The observations of SMC140 (covering central parts of the 47 Tuc globular cluster) started in 2004 and were more frequent until 2006. In total 583 epochs of this field were secured with a time baseline...
of 4.5 yr. For the rest of the LMC and SMC fields, the time baseline was almost 8 yr. The OGLE-III observing strategy was different for the LMC and SMC, because during certain sidereal hours both the SMC and the Galactic bulge can be observed. During that time the bulge fields were monitored as they are much more important for the microlensing studies. The only exceptions were during conjunctions of the bulge with Moon. Contrary, the LMC was observed all the time it was possible. The position of the LMC and SMC fields in the galactic coordinates is shown in Fig. 2.1. In the bulge fields the time baseline differs much more from field to field. The analysis was conducted in four selected fields which are characterized in Tab. 2.1 and shown in Fig. 2.2. Two additional fields were used to testing the software used.

<table>
<thead>
<tr>
<th>field name</th>
<th>R.A. Dec.</th>
<th>f</th>
<th>b</th>
<th>N_{epoch}</th>
<th>Δt</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLG134</td>
<td>17°57′38.2 ″</td>
<td>-34°12′14″</td>
<td>-3.2362</td>
<td>-4.8829</td>
<td>326</td>
</tr>
<tr>
<td>BLG160</td>
<td>18°05′52.9 ″</td>
<td>-35°26.01 ″</td>
<td>-0.8405</td>
<td>-5.5218</td>
<td>208</td>
</tr>
<tr>
<td>BLG167</td>
<td>18°03′32.8 ″</td>
<td>-31°50′15″</td>
<td>-0.5573</td>
<td>-4.8001</td>
<td>360</td>
</tr>
<tr>
<td>BLG176</td>
<td>18°06′08.7 ″</td>
<td>-31°14′55″</td>
<td>0.2313</td>
<td>-4.9995</td>
<td>355</td>
</tr>
<tr>
<td>BLG173</td>
<td>17°56′06.0 ″</td>
<td>-31°14′48″</td>
<td>-0.6135</td>
<td>-3.4997</td>
<td>786</td>
</tr>
<tr>
<td>BLG175</td>
<td>18°03′27.8 ″</td>
<td>-31°14′48″</td>
<td>-0.0478</td>
<td>-4.4975</td>
<td>384</td>
</tr>
</tbody>
</table>

Field centers are given in both equatorial coordinates (R.A. and Dec.) and galactic ones (l-longitude, b-latitude). N_{epoch} is the number of epochs collected and Δt gives total observing coverage. The results for the fields BLG173 and BLG175 are only briefly mentioned.

The fields observed during the OGLE-III project had fixed centers, and the shifts between different images of the same field resulted only from the telescope pointing errors. This makes the photometric calibration of the images easier, but at the same time makes it much harder to find a global distortion solution of the telescope (see, e.g., Bellini & Bedin, 2010). The camera was not changed or rotated (the impact of such operations on astrometric solution was investigated by Anderson et al., 2006). The transformations of the pixel coordinates from the OGLE reference images to the equatorial coordinates were based on the cross identifications with stellar positions from the 2MASS catalog (Skrutskie et al., 2006) in each of the fields. The transformations gave 120 mas rms (root mean square) per coordinate when compared to the astrometric catalog UCAC3 (Zacharias et al., 2010), which is comparable to the rms found when 2MASS is compared to UCAC3. This transformations were found before the present research was conducted and were not discussed here.
The author of this thesis conducted OGLE-III observations for 86 nights since July 2008 till the end of the project.

2.2 Standard OGLE-III reduction pipeline

The OGLE-III reduction methods were described by Udalski (2003) and Udalski et al. (2008a). Here we repeat their description with an emphasis on the aspects affecting the astrometric measurements. The reductions based on the DoPhot software (Schechter et al., 1993) are used in the present thesis only for analysis of the LMC and SMC fields. For the bulge fields the reduction was performed using software written for this purpose, what is described in detail in Sec. A.

All the OGLE-III images were corrected for the flat field and bias just after the exposition at the telescope location. Afterwards, the photometry was performed using the DIA method (Alard & Lupton, 1998; Alard, 2000; Woźniak, 2000). The resulting databases were used for most of stellar variability research, because DIA is an optimal method for the photometry in dense stellar fields. At the end of OGLE-III all the data were also
reduced using commonly used DoPHOT software. The pipeline, which divided images into subframes, ran DoPHOT, cross-matched the resulting catalogs with the reference list, and updated appropriate records in the database, was a modified version of the standard OGLE-III DIA pipeline. All these steps are described below. The purpose of this additional reduction was twofold. First, it gave the additional check for variable stars detected by DIA. Second, it allowed effortless detection of bona fide moving objects.

The standard OGLE reductions (both DIA and DoPHOT ones) depended on the reference images. These were constructed using up to 30 images taken with a good seeing conditions, low sky background and not affected by bad weather conditions, artificial lights, etc. Typically, the best seeing image was taken as the first one to the construction of the reference image and all the following images were cross-matched with that one and the appropriate grids transforming pixel coordinates were found. Next, the images were averaged starting from the best ones and taking up to 10 epochs to find pixel value on the reference image. The images were spline-resampled to the grid of first image. The spline-resampling was used because it conserves the total flux of the stars. For averaging, each subfield (corresponding to a single CCD chip of the camera) was divided into two (2180 × 2088 pixels each) or eight (1090 × 1044 pixels each) overlapping subframes depending on the stellar density. The OGLE-III reference images and resulting star catalogs in the LMC, SMC, and Galactic bulge were presented by Udalski et al. (2008b), Udalski et al. (2008c) and Szymański et al. (2011), respectively.

The way of the reference image construction presented above results in high quality images but it also has the disadvantages. Images from different epochs are taken to construct the reference one, thus, fast-moving objects are imaged at different positions. The result may resemble the galaxy seen edge-on and such an image may not be classified as a star. The main advantage of using dozens of images to construct the reference one is that the resulting image has much better signal-to-noise ratio.

The actual analysis of the science frames started with a division of each subfield into smaller subframes. Each subframe corresponded to the subframe in which the reference image was constructed. As in the case of the reference image construction, the pixel coordinate transformation was found using the brightest stars and performed using the spline method. On the subframes of the image prepared in such a way the DoPHOT (Schechter et al., 1993) software was run. The software was modified by Dr. hab. Michal
Szymański and Prof. Andrzej Udalski in order to run it on larger images and perform calculations in the double-precision floating point arithmetic (original software used the single-precision arithmetic). The position and brightness of each star is found by fitting the analytical PSF which is a Taylor expansion series of the two-dimensional Gaussian function. The coefficients defining the PSF depend on the position on the chip. The stellar catalog resulting from DoPHOT was cross-matched with the list of stars found in the reference image with the matching radius of 1.9 pixel = 0\textquotesingle 49. It was possible that one star measured on a given image was associated with two records on the reference list. The cross-match list was added to the database of results similar to the one described by Szymański & Udalski (1993). The database user has only access to the positions measured using DoPHOT and the transformation grids were not recorded.

Such a method of data reduction was efficient computationally and easy to apply but it was not optimal for astrometric measurements. Most importantly, DoPHOT aims at good quality photometry, not necessarily good astrometry (see discussion in Anderson & King, 2000). The above mentioned procedure uses not only DoPHOT to measure the positions of stars. The grids transforming pixel coordinates are calculated using the bright stars positions measured by the sfind software which is a part of the DIA package (Woźniak, 2000). sfind finds the stellar centroids using parabola fitting to marginal sums\(^1\) in a 3 \times 3 pixel subarray. The results of sfind and DoPHOT run on exactly the same image may be different. To check this, we calculated the differences between positions returned by DoPHOT on the resampled image and catalog positions of stars. These differences were averaged and examined. It turned out that the highest difference was 0.1 pixel. We inspected the image and it turned out that this particular exposition had seeing better than the average, but the PSF was elongated and DoPHOT divided each star into two separate objects which affected the measured positions.

These are not the only disadvantages of the standard OGLE-III DoPHOT reductions. The transformation grids were calculated using all the bright stars including the HPM objects as well as very blue and very red stars. The positions of stars which significantly differ in color are affected by the differential refraction. Also the procedure of resampling was not optimal because the spline resampling does not conserve the relative stellar centroids. At the very end the cross-match was performed with a constant radius which was too large

\(^1\)The X-direction marginal sum is a sum of pixel values down the columns of the image subarray. Similarly for Y-direction, the sum is calculated across the rows.
for the bright stars and was not optimal for the HPM stars.

Because of all these failures, our reductions of the bulge data were performed in a
different way. We measured centroids in raw images before any grids were calculated.
The software used was optimized for accurate measuring the stellar centroids. All the
consecutive data reduction steps aimed at the best possible accuracy of proper motions
and parallaxes.
Chapter 3

Analysis of fields toward the Magellanic Clouds

In these chapter we will present how the proper motions and parallaxes of stars observed in the OGLE-III LMC and SMC fields were calculated. The following analysis is concentrated on the stars that belong to the Galaxy and lie in the foreground of the Magellanic Clouds. Since most stars observed in these fields are members of the LMC and SMC, they are used to construct reference frames for measured stellar positions. The only exception is the field SMC140 in which great majority of stars belongs to the globular cluster 47 Tuc. The parts of the SMC131, SMC136, and SMC137 fields are also located within the tidal radius of the 47 Tuc which is 429, according to Kiss et al. (2007). In these fields, the number of the SMC stars is comparable to the number of 47 Tuc ones. Thus, the term “background” denotes either the LMC, the SMC, or 47 Tuc (only SMC140 field), and the proper motions will be tied to the background stars. We also present the measurement of the absolute proper motion of the LMC and the relative proper motion of the 47 Tuc and the SMC.

3.1 Astrometric reductions

The reduction started with the stellar centroid measurements using the DoPhot software as described in Sec. 2.2, which were obtained from the OGLE databases. For each epoch and each star the uncertainty of centroid fitting ($\sigma_{PSF}$) was found using the formula derived by Kuijken & Rich (2002):

$$\sigma_{PSF} = \frac{0.67 \cdot \text{FWHM}}{S/N}$$

(3.1)
where FWHM is the Full Width at Half Maximum of the stellar profile for a given exposition and S/N denotes the signal-to-noise ratio of the stellar flux. The values of FWHM were measured for each exposition. To do this we ran DAOPHOT photometric software (Stetson, 1987) on each image and cross-matched the output of FIND procedure with a catalog of bright stars in the reference image. Having this we could measure the FWHM for all the images. In some subfields the standard OGLE-III pipeline failed to measure the FWHM. An example can be LMC156.7 which is one of the subfields with the lowest stellar density but there are a few overexposed stars. The S/N was estimated using the uncertainty of brightness ($\sigma_m$ [mag]), as returned by the DoPHOT. The standard photometric formula was used:

$$\sigma_m = \frac{1.086}{S/N}$$  \hspace{1cm} (3.2)

We note that $\sigma_{\text{PSF}}$ reflects only the uncertainty related to the finite number of ADU and the fact that atmosphere blurs the images of stars. If the positions of the star from different images are compared, one has to take into account not only $\sigma_{\text{PSF}}$ but also the uncertainty of fitting grids between the images ($\sigma_{\text{grid}}$). Thus, in further steps of the data reduction, the $\sigma_{\text{PSF}}$ is square added to the grid uncertainty found using bright stars:

$$\sigma_{\text{cent}} = \sqrt{\sigma_{\text{PSF}}^2 + \sigma_{\text{grid}}^2}$$  \hspace{1cm} (3.3)

what gives the uncertainty of the stellar centroid in a given grid ($\sigma_{\text{cent}}$), and is used to derive the uncertainties of astrometric parameters in the $\chi^2$ minimalization. The $\sigma_{\text{cent}}$ is dominated by $\sigma_{\text{PSF}}$ for faint stars, while for bright stars the $\sigma_{\text{grid}}$ is dominating.

The next step was to divide the list of stars for each subfield in subframes of either 2180 $\times$ 2088 or 1090 $\times$ 1044 pixels, corresponding to subframes in which data were separately reduced. All the following steps, which aimed at finding proper motions and parallaxes, were performed separately for stars in each subframe.

The process of proper motion and parallax estimation was iterative. The astrometric transformation of centroid measurements is based on the list of good stars which we define as stars brighter than $I = 18$ mag, with color information and insignificant parallax. The color information for these stars is important because the differential refraction coefficient is a linear function of the stellar color. In the consecutive steps described below, we tried to correct: the list of images used, the list of good stars used, the $r$ vs. $(V - I)$ relation,
the corrections for centroids and, the $\sigma_{\text{grid}}$ values.

In the first step of the proper motions calculations, for every star brighter than $I = 18$ mag for which $(V - I)$ color was available, we fit two models. One has five free parameters: $\alpha_0$, $\delta_0$, $\mu_\alpha$, $\mu_\delta$, and $r$. The other model has $\mu_\alpha$ and $\mu_\delta$ fixed to 0—there are only three free parameters—and was performed in order not to overfit the data. This and all the following model fits are performed using the Singular Value Decomposition (SVD) method.\footnote{Singular Value Decomposition is the method of a factorization of a matrix. It is capable of dealing with matrices which are numerically close to singularity (Press et al., 1992).} We note that the number of equations is twice larger than the number of epochs for a given star. From the Eqs. 1.1 and 1.2, it is clear that the set of equations on which parameters are found is naturally divided in two subsets which have only two common parameters: $r$ and $\pi$ (the latter is not used in this step of iteration). In this step, we assumed $\sigma_{\text{grid}} = 0$ and the uncertainties of parameters fitted were estimated using the covariance matrix returned by the SVD method. From two fits we kept the results of the one with $\mu_\alpha = 0$ and $\mu_\delta = 0$ if the resulting reduced $\chi^2$ was larger than 1.1 times the resulting reduced $\chi^2$ for fit with free parameters $\mu_\alpha$ and $\mu_\delta$. If the SVD did not found very large ratio of the two eigen values and the maximum number of iterations was not approached as well as resulting $\mu$ was smaller than 20 mas/yr, the star was added to a list of good stars.

The limit on maximum proper motion was used in order to assure that only stars with negligible parallax are on the list of good stars. Fitting a model without parallax (as we performed for good stars) for a star with a significant parallax should result in systematic error in estimated $r$.

Some of the frames collected during the OGLE-III observations were taken under bad weather conditions, had very large shifts compared to the field center or had some problems in data reduction process. In order to make our results more reliable, we removed them by imposing the limits on the number of good stars detected. The results from the first step of iterations were also used to find the first approximation of the $r$ vs. $(V - I)$ relation.

The role of the second step of iteration was to remove stars that had very noisy measurements from the list of good stars. Those were typically the ghost stars that were found in the neighborhoods of the overexposed stars. Each good star was fitted with the model containing either two ($\alpha_0$ and $\delta_0$) or four ($\alpha_0$, $\delta_0$, $\mu_\alpha$, and $\mu_\delta$) free parameters depending on the best model chosen in the first iteration. The differential refraction coefficient was fixed at the value resulting from the relation to color found in the previous step. If the fitting
procedure failed or resulting proper motion was greater than 20 mas/yr limit, we removed such a star from the good star list. To derive the corrections to the positions, we calculated expected positions for every good star and every image separately. The expected positions were subtracted from the observed ones and the resulting residua were averaged for each epoch. The opposite of the average residuum was taken as an estimate of the correction to position. The $rms$ of the residua was taken as an estimate of the position uncertainty.

In the next step of iteration the two models were fitted once more—one with the proper motion fixed to 0 and one with $\mu_\alpha$ and $\mu_\delta$ set as free parameters. The second one was accepted if the resulting reduced $\chi^2$ was smaller than 0.95 times the reduced $\chi^2$ for model with $\mu_\alpha = \mu_\delta = 0$. All the stars with $\chi^2 > 10$ or $\mu > 10$ mas/yr were removed from the list of good stars. The results were used to find new linear relation between $r$ and $(V - I)$.

The fourth step of iteration aimed at calculating the corrections for measured positions with the mean proper motion of good stars fixed to 0. To do this, we once more fitted time-series centroids of each good star with a model which either was kept fixed to 0 or not (depending on results from previous iterations). The $4\sigma$-clipping was performed to find mean proper motion of good stars. Next, the residua were calculated for each good star with the model resulting from the last fit and subtracted mean values of the proper motion derived. The residuals were averaged for each epoch and their opposite was taken as a correction for positions. Also the corrections for centroid uncertainties were derived in this step. The $rms$ of proper motions of background stars was a measure of systematic uncertainties. Statistical uncertainties resulted from the covariance matrix found during the fitting process for each star.

The last iteration involved the final fitting for all the stars. The centroids and their uncertainties were corrected using the values found above. For each star four or eight models were fitted depending on the availability of color information: with $\pi$ as a free parameter or fixed to 0, with $r$ as a free parameter of fixed at the value resulting from the color of the star, and using all the epochs or only those with seeing better than 4.5 pixel = 1.17. Fitting the models with $\pi$ set as a free parameter in 75% of cases resulted in matrices which could not be inverted.
3.2 High proper motion stars

3.2.1 Selection

The results were first used to select the HPM stars, which are hereinafter defined as stars with $\mu > 100$ mas/yr. This limit is chosen arbitrarily; however, we note that stars with $\mu > 126$ mas/yr moved during the time span of the OGLE-III observations of the Magellanic Clouds far enough to produce two separate records in the catalog of objects found on the reference frame. To select HPM stars we, analyzed the list of all measured proper motions and chose the stars for which at least one of the fitted models gave $\mu > 95$ mas/yr. The limit was lowered by 5 mas/yr in order to have the complete list of stars with $\mu > 100$ mas/yr, as a few outlying points might have affected the proper motion value. In many cases the images of the candidate HPM stars are elongated on the OGLE reference frames and they could have been either detected as a few separate objects or not detected at all because of the unusual intensity profile. Later-on we will show that the second possibility happened at least once.

We had found many artifacts in our preliminary list of HPM stars. They had only a few dozen of epochs and proper motions found were only caused by chance-alignments of a small number of points. Some artifacts had very large proper motion uncertainties (above 10 mas/yr) and were caused by neighboring overexposed stars. Many such artifacts were visually verified and the rest were removed based on the number of epochs, proper motion uncertainty and $\chi^2$ of the model fitted.

The list of candidates was compared to previously published catalogs of the HPM stars observed towards the Magellanic Clouds, which were based on the MACHO (Alcock et al., 2001) and the OGLE-II (Soszyński et al., 2002) surveys. These two catalogs contained 80 unique stars with $\mu > 95$ mas/yr. Our list of candidates contained almost all of these stars, i.e., 76 objects. Two out of four missing objects are saturated on the OGLE-III reference images and thus could not be measured in the present analysis; their IDs given by MACHO are 2.4668.10 and 5.5613.1633. For the third missing object (LMC_SC8 359715), Soszyński et al. (2002) presented the proper motion of $125.6 \pm 4.0$ mas/yr. This object is present in the two adjacent fields of the OGLE-III survey. Our final catalog gave in both these fields consistent results: $\mu = 85.6 \pm 0.7$ mas/yr for LMC100.595501 and $\mu = 85.4 \pm 0.9$ mas/yr for LMC101.825901. We note that there is a star fainter by $\approx 1.8$ mag in the $I$-band on
the OGLE-III reference image. The distance between the stars is 0″73, which translates to 1.7 pixel in the OGLE-II images. This additional star might have affected the proper motion measured by Soszyński et al. (2002) more than measured here using images with better angular resolution.

The last star from Alcock et al. (2001) and Soszyński et al. (2002) with \( \mu > 95 \) mas/yr which was not found on our list of candidate HPM stars has the MACHO identifier 206.16886.2221 and the OGLE-II identifier SMC_ SC10 57257. Its proper motion is 365.01±0.32 mas/yr. We examined the OGLE-III reference image at the expected position of that star. It turned out that the moving object produced a few blended centroids on the reference image and there were two additional stars very nearby. The resulting image was classified by DoPHOT as a diffuse object and thus not included in the standard OGLE-III reductions. We obtained the centroids for that star from the database of the raw DoPHOT results (i.e., before they were cross-matched to known stars). The star was given the OGLE-III identification: SMC110.5.999999 and it was further analyzed in the same manner as other objects. The comparison of our list of candidate HPM stars with the MACHO and the OGLE-II lists showed that 76 out of 77 objects that could be found in the OGLE-III data were found (excluding 2.4668.10, 5.5613.1633, and LMC_ SC8 359715). These translates to a very high completeness of 99%.

Most of the candidate HPM stars were split into a few records in the database of a given field, due to the duplicate records were removed and for each object all the centroids close to the candidate HPM were retrieved and analyzed together. Next, centroids for each candidate HPM star were examined in detail (see Fig. 3.1). The points which were taken in bad weather conditions were removed, especially when bad seeing caused merging of the images of the HPM with a nearby star. We obtained the clean sample of centroids for each candidate HPM star in each subchip separately.

The adjacent OGLE-III fields are overlapping and some objects are present in two, three, or four overlapping fields. We checked if our candidate HPM stars were present in two fields. If a number of measurements in both fields was comparable, we performed an additional fit, which used the data from both fields. In order to account for possible differences in astrometric solutions and refraction coefficients in both fields, the J2000.0 equinox and \( r \) coefficients were set as separate free parameters. The proper motions and parallaxes were kept the same during fitting process. Thus, for the stars present in two
3.2 High proper motion stars

![Graph showing proper motion data with color-coded scale](image)

Figure 3.1: Example plot of verification image. Relative differences in R.A. and Dec. are shown. The bar shows the scale of the color coded epoch of observation HJD $- 2450000$. The star on the right-hand side has significant proper motion, while the one on the left-hand side in not moving. Typically, the plots contained only one star and thus covered smaller sky area.

fields we had a model with nine free parameters ($\alpha_{0,1}$, $\alpha_{0,2}$, $\delta_{0,1}$, $\delta_{0,2}$, $\alpha_1$, $\delta_1$, $\mu$, $\delta$, and $\pi$). The number of equations was twice the sum of number of epochs in both fields.

When all the HPM candidates had their final models fitted in either one or two fields, we compiled the final HPM list by removing the stars with $\mu < 100$ mas/yr. Two example fits are presented in Fig. 3.2. We were left with 549 HPM stars, of which 369 are observed towards the LMC and 180 towards the SMC.

The final list of HPM stars with two additional stars is presented in electronic form to the astronomical community. The list can be accessed via anonymous FTP site:


The main file is ident.dat, which contains columns: OGLE-III identifier (Udalski et al., 2008b,c), J2000.0 equatorial coordinates, proper motion with statistical and systematical uncertainties, parallax, $I$-band magnitude, $(V-I)$ color and luminosity class. The brightness and color given there differ from the ones presented by Udalski et al. (2008b,c). These
Figure 3.2: Two example model fits. The ordinate axis show difference between current position and the mean one in equatorial coordinates. The differential refraction effect was subtracted. The model parameters ($\mu_\alpha$, $\mu_\delta$, $\pi$) are (10.1, 394.6, 91.3) for LMC194.6.41 and (-98.2, 24.8, 19.1) for LMC129.1.16051.
3.2 High proper motion stars

differences are caused by incorporation of additional correction for transmission differences between the standard filter and the one used by the OGLE-III. This correction is negligible for stars with \((V - I) < 1.5\) mag and arises to 0.1 mag for \((V - I) = 4\) mag (Szymański et al., 2011). The cross-match between \(I\) and \(V\)-band was performed for each star individually, what, in some cases, changed the \((V - I)\) color significantly. The luminosity class is indicated only for WDs (21 definite and 23 candidates) and one subdwarf (see Sec. 3.2.5). Cross-matches with MACHO, OGLE-II, and SPM4 catalogs as well as remarks are given in the separate files. The finding charts are presented for all objects. The directory contains the file describing the catalog content and structure of the files in detail.

3.2.2 Results

The highest proper motion found in this study was \(\mu = 722.19 \pm 0.74\) mas/yr for a star LMC198.4.97 (\(\pi = 29.4 \pm 1.7\) mas). This is 14.3 times smaller value than the proper motion of the Barnard’s star. The largest parallax found is \(\pi = 91.3 \pm 1.6\) mas/yr for LMC194.6.41 (\(\mu = 394.71 \pm 0.51\) mas/yr). Model fit for this star is shown in the upper panel of Fig. 3.2. This star is 8.4 further than the Proxima Centauri. The RECONS survey, which aims at cataloging nearest stellar systems, lists 100 stellar systems within the 6.6 pc radius\(^2\). Assuming constant stellar density in the solar neighborhood, we can estimate that LMC194.6.41 is \(\approx 470^{th}\) stellar systems closest to the Sun. Proper motions and parallaxes of LMC198.4.97 and LMC194.6.41 were not known before.

We used the fastest HPM stars to check if they could act as gravitational microlenses for more distant stars. The surroundings of each star were visually inspected with overplotted the direction of the HPM star proper motion. In none of the fields examined we could see a possible HPM-background star alignment. The lack of predicted microlensing events is not surprising, as the probability of these events strongly depends on stellar background density. Similar examination of the proper motions derived in the Galactic bulge fields should be more fruitful.

3.2.3 Completeness

The comparison of the candidate HPM stars with lists presented by Alcock et al. (2001) and Soszyński et al. (2002) was already presented. The number of the HPM stars that we

\(\text{http://www.chara.gsu.edu/RECONS/TOP100.posted.htm}\)
found which are inside the sky area covered by the OGLE-II and the MACHO projects was 79 and 271, respectively. These surveys presented, respectively, 62 and 26 of them. The much larger number of objects found here coupled with a small number of objects missed (1 out of 77) shows that completeness of our HPM list is very high.

There are other stellar proper motion catalogs which cover the sky area observed by the OGLE-III survey. One of these catalogs is the fourth installment of the Yale/San Juan Southern Proper Motion Catalog (SPM4) described by Girard et al. (2011). We selected the stars with $\mu > 100$ mas/yr that are located in the OGLE-III Magellanic Clouds fields and have $V$-band brightness similar to the stars studied here, i.e., between 14 mag and 21 mag. The total number of stars selected was 7786. In order to check the reliability of the SPM4 proper motions, we retrieved time-series astrometry from raw DoPHOT results database for randomly selected 100 stars. Every object was queried with a 3$''$ radius. The plots of $\alpha \cos \delta$ vs. $\delta$ with color-coded epoch of observation were visually examined (see example plot in Sec. 3.1). Only eight out of hundred plots showed clearly moving objects. We once more examined them in detail. Two out of these objects are HPM that are in our list, while the rest have $\mu < 50$ mas/yr. Our conclusion is that the SPM4 is much less reliable compared to our HPM star list in the dense stellar region of the Magellanic Clouds.

An initial check done with the UCAC4 catalog\(^3\), similarly to SPM4, revealed thousands of HPM stars in our fields. The large number of HPM stars selected suggests high false positive rate in the UCAC4 catalog. We note that the region of Magellanic Clouds is typically excluded when HPM stars are investigated (e.g., Finch et al., 2007).

Another verification of the completeness of our list can be done internally. For 549 HPM stars there are 582 entries in the OGLE-III photometric maps which have significant number of measurements, i.e., 31 stars are present in two adjacent fields. All 31 stars were found independently in each of the fields. This is another indication of high completeness of our HPM star list.

We also wanted to check if completeness of our HPM star list depends on the position on the sky and is affected, e.g., by the high stellar density. Figure 3.3 shows the sky projection of the HPM stars found. No obvious dependencies can be seen except the lack of HPM stars in the Tarantula Nebula (field LMC175; middle left of the upper panel) and their small number in the very center of the SMC. The mean number of the HPM stars per area

\(^3\text{http://cdsarc.u-strasbg.fr/viz-bin/Cat?1/322}\)
covered by one OGLE-III field is 3.4 in the LMC fields and 4.6 in the SMC fields. These numbers can be compared with the predictions of the Besançon Galactic model (Robin et al., 2003) with default parameters. The number of the predicted HPM stars in the $I$-band brightness range of 12.8–20.2 mag for the centers of the LMC and the SMC was 4.3 and 4.7, respectively. Thus, the measured values are 79% for the LMC and 98% for
the SMC of the predicted ones.

3.2.4 Accuracy

None of our HPM stars is bright enough to be found in catalogs of the Hipparcos satellite. In order to externally check our uncertainties, we compared the derived proper motions and parallaxes with the ones previously published by Alcock et al. (2001) and Soszyński et al. (2002). Both these works as well as our analysis were based on relatively short surveys of high stellar density regions. In such cases the stellar blending may be the main problem affecting measured proper motions. The number of objects common with the MACHO and OGLE-II lists are 26 and 62, respectively. The 3σ-clipped difference of $\mu_\alpha*$ and $\mu_\delta$ had rms of 5.9 mas/yr when we compared with the MACHO list and 3.3 mas/yr when we compared with the OGLE-II list. Alcock et al. (2001) gave only typical uncertainty of their proper motions—roughly 3.5 mas/yr per coordinate. Based on uncertainties estimated by us and Soszyński et al. (2002), we estimated the expected rms of 1.7 mas/yr, i.e., twice smaller than measured.

There are 11 stars with parallaxes estimated by us and by Soszyński et al. (2002). Our results turned out to be on average greater by $2.5 \pm 1.0$ mas. We note that our analysis of parallaxes preceded by removing nearby stars from the stars to construct the reference grid. Thus, the zero point of our parallaxes should be closer to the absolute frame than the zero point of Soszyński et al. (2002) parallaxes. This effect should be responsible for at least a part of the mean difference in parallaxes.

3.2.5 Physical parameters

The parallaxes were measured for 385 HPM stars with significance greater than 3σ. This allowed the calculation of the absolute magnitudes in the $I$-band:

$$M_I = I + 5 \log \pi + 5$$

(3.4)

Extinction is negligible for stars discussed here. The Hertzsprung–Russell diagram—a plot of $M_I$ vs. $(V - I)$—is shown in Fig. 3.4. The errorbars of $M_I$ include uncertainty of both $\pi$ and $I$. For stars in the bottom right part of the plot the $(V - I)$ color is purely known because of the faintness of this stars in the $V$ band. The gray lines present Holberg &
3.2 High proper motion stars

Bergeron (2006) models for pure Hydrogen WDs with logarithm of surface gravity ($\log g$, where $g$ is expressed in cm/s$^2$) between 7.0 and 9.5. Based on these lines, 21 WDs were selected. They are marked with light blue triangles on Fig. 3.4. Comparison of the Holberg & Bergeron (2006) models with positions of the WDs identified gave an estimates of effective temperatures ($T_{\text{eff}}$): the two bluest and brightest WDs (LMC163.256430, SMC110.823710) have $T_{\text{eff}} > 15000$ K, the group of WDs with 12 mag < $M_I$ < 15 mag has 4700 K < $T_{\text{eff}}$ < 12000 K, and the five faintest and reddest WD have $T_{\text{eff}} < 6000$ K. Between the clearly seen main sequence (MS) and the WDs there is a dozen or so stars that are candidate subdwarfs. Absolute brightness is poorly known for most of them except one star with $(V-I) = 1.51$ mag and $M_I = 12.1$ mag. This star is SMC110.599999, which was not on our candidate HPM list but was added because Alcock et al. (2001) and Soszyński et al. (2002) found this star.
Figure 3.5: Color-magnitude diagram for the HPM stars. Black and light blue symbols indicate the same stars as in Fig. 3.4. Dark blue symbols mark candidate WDs while the rest of stars, without parallax measured, is shown using red circles.

For 30% of the HPM stars the parallax was not found. To reveal their luminosity class we plotted the CMD for all HPM stars, which are shown in Fig. 3.5. The light blue and black symbols mark the same stars as on Fig. 3.4. The stars without parallaxes measured are shown using dark blue symbols if they are bluer and fainter than the locus of WDs identified on Fig. 3.4, or red circles otherwise. The 23 objects marked with dark blue symbols are candidate WDs. We explain the stars marked by red circles as MS stars which are farther away than the stars with measured parallaxes.
3.3 Catalog of stellar proper motions

3.3.1 Catalog construction

The construction of the catalog was based on the proper motions and parallaxes derived earlier as described in Sec. 3.1.

The overexposed stars produce many ghost images and strongly affect nearby objects. The saturation limit in the OGLE-III $I$-band images is between 11.5 and 12.7 mag and depends on the number density of stars. We removed the stars brighter than 11.5 mag from our catalog and all the nearby ones. The list of such stars was taken from the shallow survey conducted in parallel to the OGLE-III project (Ulaczyk et al., 2012, for 90\% of the LMC fields) and Deep Near Infrared Survey of the Southern Sky catalog (Cioni et al., 2000, for the remaining LMC fields and all SMC ones). We found a linear relation between $I$-band magnitude of a bright star and a distance in the sky within which other stars were affected. This radius was 1′ for $I = 5.9$ mag and 0′ for $I = 11.5$ mag. This procedure reduced the sky-area analyzed by 0.18 and 0.052 square degrees for the LMC and SMC, respectively (0.005 square degrees in the SMC140 field).

The other source of contamination are photometrically variable stars. Their changing flux shifts the centroid, if the star is blended, and affects the centroids of closest stars. This effect most severely acts on large amplitude variables, e.g., Mira-type stars. It is easily seen in blended eclipsing binaries for which centroids clump in two groups: the first of them contains epochs of maximum brightness, while the second one—during the eclipses. If the ingress and egress are short compared to the orbital period, the two groups of points can be fully resolved. In such situations fitting a model defined by Eqs. 1.1 and 1.2 to the observed centroids would give wrong results. Hence, we analyzed the variable stars separately (see Sec. 3.3.8) and remove all the stars which are within 1″ radius from each variable. This decreased the sky-area analyzed by 0.036 and 0.006 square degrees for the LMC and the SMC fields, respectively. The list of variable stars was taken from the OGLE-III Catalog of Variable Stars which includes the classical, type II and anomalous Cepheids, RR Lyr variables, long period variables, R CrB variables, δ Sct stars, eclipsing binary systems, and the so-called double periodic variables (last four types of variable stars are from the LMC fields only). The OGLE-III Catalog of Variable Stars was presented in a series of papers (e.g., Soszyński et al., 2008, 2010; Poleski et al., 2010; Graczyk et al., 2011).
The last problem we faced during catalog construction was the influence of outlying measurements. Even a few badly cross-matched centroids can significantly change the results of the least-squares fitting. The typically used procedure in such cases is a $\kappa \sigma$-clipping method. We did not find the method which removes the outlying measurements and at the same time does not affect the results for astrophysically important objects (e.g., stars with the highest proper motions or close CPM binaries). Instead of removing the outlying measurements before the final fitting is performed, we decided to include in the catalog only objects with reliably derived proper motions.

![Chi-squared vs. I-band brightness](image)

**Figure 3.6:** Reduced $\chi^2$ vs. $I$-band brightness diagram. Dashed lines indicate borders of brightness bins in which stars were selected. The stars that passed the visual verification are marked using gray circles and the ones that failed—using black crosses. The black line ($\chi^2(I)$ criterion) is used to automatically select stars with reliable proper motions.

The selection of stars with reliable proper motions was based on the examination of the plots similar to the one presented on Fig. 3.1. For stars with $\mu > 20$ mas/yr one in most cases can easily recognize that either the star is moving or the proper motion found is caused by a small number of epochs, large scatter of data-points, etc. We divided the data into magnitude bins: $I \leq 13$ mag, 13 mag $< I \leq 14$ mag, \ldots, 18 mag $< I \leq 19$ mag, and 19 mag $< I \leq 19.5$ mag. In each of the bins $\approx 500$ stars were selected for the visual
examination. Figure 3.6 presents all the verified stars in the plot of $\chi^2$ (reduced) of the model fitted vs. $I$-band brightness. The gray circles mark the stars which were positively verified, while dark crosses mark the stars which were negatively verified. There is a clear division of the two groups on the plot which we used to define a region in which stellar proper motions are highly reliable. The division line is shown in black in Fig. 3.6 and we call it the $\chi^2(I)$ criterion. The line is vertical at $I = 19$ mag because for the fainter stars the relative number of stars with reliable proper motions is very low.

Table 3.1: Statistical properties of the $\chi^2(I)$ criterion applied

<table>
<thead>
<tr>
<th>magnitude bin</th>
<th>reliability</th>
<th>completeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 15 mag</td>
<td>95.1%</td>
<td>93.9%</td>
</tr>
<tr>
<td>13 – 14 mag</td>
<td>96.9%</td>
<td>92.6%</td>
</tr>
<tr>
<td>14 – 15 mag</td>
<td>97.7%</td>
<td>93.2%</td>
</tr>
<tr>
<td>15 – 16 mag</td>
<td>98.9%</td>
<td>93.6%</td>
</tr>
<tr>
<td>16 – 17 mag</td>
<td>98.1%</td>
<td>95.1%</td>
</tr>
<tr>
<td>17 – 18 mag</td>
<td>98.2%</td>
<td>98.2%</td>
</tr>
<tr>
<td>18 – 19 mag</td>
<td>93.9%</td>
<td>92.0%</td>
</tr>
</tbody>
</table>

In Tab. 3.1 we present the reliability of the $\chi^2(I)$ criterion, i.e., the ratio of the number of stars that passed visual examination to the number of stars that fulfilled the criterion. Last column of this table presents the completeness, i.e., the number of stars that fulfill the criterion to the number of stars that passed visual examination. Except the faintness bin the reliability is above 95%. In all cases the completeness is above 90%. The $\chi^2(I)$ criterion was derived using stars with $\mu > 20$ mas/yr but we used it to automatically select all the stars with reliable proper motions.

The final catalog of over 6.2 million stellar proper motions is available via anonymous FTP site:

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The main part of the catalog provides for each star: OGLE-III identifier, J2000.0 equinox coordinates as well as the $I$-band brightness and $(V-I)$ color (these params are the same as presented by Udalski et al., 2008b,c, except for HPM stars), proper motion in both coordinates with statistical and systematic uncertainties, total proper motion, parallax and its uncertainty, differential refraction coefficient and its uncertainty, $\chi^2$ per degree of freedom for the model used, $\chi^2(I)$ value, number of data points used for fitting, flags showing objects which were visually verified and ones for which only data from the best
seeing images were used. Parallaxes are presented only for stars with at least 200 epochs in
the LMC fields and at least 300 epochs in the SMC fields. Over 110 000 stars fulfilled these
criteria. To allow statistical analysis of the selected groups of stars, we did not put any
lower limit on significance of the presented parallaxes and even negative results were given.
These data are presented both in 1256 separate files for each of the OGLE-III subfields
as well as in one archive containing the whole catalog. Additional files describe the CPM
binaries (Sec. 3.3.5), variable stars (Sec. 3.3.8), and remarks to selected objects.

3.3.2 Completeness

The completeness of our catalog strongly depends on the proper motion value. As it was
presented earlier, it is very high for stars with $\mu \geq 100$ mas/yr. We visually inspected all
the stars with $70$ mas/yr $\leq \mu < 100$ mas/yr, no matter if they fulfilled the $\chi^2(I)$ criterion
or not. Also randomly selected stars with $\mu \geq 20$ mas/yr were examined, what made their
list more complete. The catalog contains 19,807 stars which were visually examined. For
the rest of the stars, the estimated completeness is given in Tab. 3.1 as a function of the
$I$-band brightness.

As an additional check for the catalog completeness, we examined the distribution of
disk MS stars, halo MS stars, WDs, and stars without color information (see Sec. 3.3.4)
as a function of $\mu$. It is shown in Fig. 3.7 for stars with $\mu > 30$ mas/yr. The relations
shown have almost constant slope, except for the WDs group for which the slope changes
at our limit for HPM stars of $\mu = 100$ mas/yr. We did not find the explanation of the
slope change. In the SMC fields the number of objects with the highest proper motions
(\(\mu > 300\) mas/yr) is smaller than expected from extrapolation of the trend seen at smaller
values.

The relations for all stars presented in Fig. 3.7 were fitted in the range $35$ mas/yr $<
\mu_0 < 100$ mas/yr for the LMC and SMC fields separately:

$$\log n_{\text{LMC}} (\mu > \mu_0) = -2.80 \log \mu_0 + 8.16$$ (3.5)

$$\log n_{\text{SMC}} (\mu > \mu_0) = -2.68 \log \mu_0 + 7.62$$ (3.6)

These relations were used to statistically quantify if the pairs of stars with similar proper
motions and lying close to each other are chance alignments or belong to the CPM binaries.
3.3 Catalog of stellar proper motions

Figure 3.7: Number of objects with $\mu$ higher than a given limit of $\mu_0$. The relation for all objects is shown (dashed dark gray line) and separately for: disk MS stars (black solid line), halo MS stars (dashed line), WDs (light gray line), and stars without color information (dotted line). The results for the LMC and SMC fields are shown in left and right panel, respectively.

(Sec. 3.3.5).

The total number of foreground Galactic stars in the catalog was found by counting stars with the significance of the proper motion higher than $5\sigma$. Out of 6.2 million stars in the catalog 440 000 satisfied this criterion.

3.3.3 Accuracy

The statistical uncertainties of the proper motions are shown in Fig. 3.8 separately for a dense and sparse stellar field. For bright stars the uncertainties in the dense field are smaller because of larger number of stars used in the frames alignment. In a dense field, one can see the group of stars fainter than 18 mag with uncertainties larger by $\approx 30\%$ than the rest of the stars. Those are the objects that passed the $\chi^2(I)$ criterion only when the best seeing epochs were used in the fit. Down to the brightness of 18.5 mag the uncertainties are smaller than 0.5 mas/yr.

The uncertainties of parallaxes are down to 1.6 mas. Systematic offsets might be caused
by unaccounted second-order effects in differential refraction. The grids were found using the stars with \( \mu < 20 \text{ mas/yr} \). In these sample selected stars may have parallaxes similar to the smallest uncertainties.

### 3.3.4 Physical parameters

Figure 3.9 presents the Hertzsprung-Russell diagram for 12,350 stars with \( \pi/\sigma_\pi > 3 \). The main sequence (MS) is clearly visible. We note that very few MS stars in the solar neighborhood are bluer than \((V-I) \approx 0.7 \text{ mag}\), what causes the sharp cut at this color. The gray lines present theoretical WD positions from Holberg & Bergeron (2006). Around 270 objects are in the part of the diagram enveloped by these models. Subdwarfs, which are between 2 and 4 mag fainter than the MS, can also be selected.

The luminosity class and population to which a given star belongs can be found using a plot of the Reduced Proper Motion (RPM) vs. color—the so-called RPM diagram. RPM
is defined as

\[ H_I = I + 5 \log \mu \]  

(3.7)

If all the stars had the same tangential velocities, it would be the same as Hertzsprung-Russell diagram with ordinate scale added to constant value. This constant depends on the tangential velocity. The RPM diagram is a useful diagnostic tool even though the assumption that all stars have the same tangential velocities is not fulfilled. We present the RPM diagram constructed using stars with \( \mu > 30 \text{ mas/yr} \) in Fig. 3.10. The distribution of stars on that plot is similar to the one in the RPM diagrams constructed by other authors (e.g., Chanamé & Gould, 2004). The inclined lines separate WDs, halo MS stars and disk MS stars and were determined to give the best discrimination between different stellar populations.
3.3.5 Common proper motion systems

There are many applications of CPM binaries, especially if large and homogeneous data-set is used (Chanamé, 2007). We searched for CPM binaries among stars with $\mu > \mu_{\text{lim}} = 30 \text{ mas/yr}$. This cut allowed the detailed case-by-case verification of each CPM binary. The OGLE-III fields were overlapping, thus, we started with preparing the list of unique stars with $\mu > \mu_{\text{lim}}$. It contained 10,405 stars in the LMC fields, and 4,378 stars in the SMC fields. All these stars were visually inspected both to check the reliability of our results and to find the closest CPM companions—at separations of $\approx 1''$. For such closely lying stars the mean magnitudes might be inaccurately measured. In some cases we manually removed the points outlying most and derived the proper motions and parallaxes ones more.

Our method of the CPM selection is similar to the one of Chanamé & Gould (2004). Recently, Tokovinin & Lépine (2012) presented similar analysis using Hipparcos data. We note that simple comparison of differences in $\mu_\alpha$ and $\mu_\delta$ with uncertainties of these quantities (as used by, e.g., Dhital et al., 2010) is not adequate for data analyzed here. The
proper motions uncertainties in our catalog are below 1 mas/yr and may be smaller than the proper motions resulting from the orbital motion of the binary.\footnote{For a binary system with a total mass of 1\textit{M}_\odot and separation of 500 A.U. seen face-on at a distance of 100 pc (angular separation of 5”), the orbital motion causes the proper motion of 2.8 mas/yr.} Below we describe our method of CPM binaries selection which is based on the comparison of the candidate CPM binaries parameters with unrelated pairs. Then, we verified positions of CPM binaries components on the RPM diagram. We end with statistical analysis and the description of the most interesting systems.

For every star with \( \mu > \mu_{\text{lim}} \), we found companions closer than 1500\''\(\mu\). The great majority of the pairs selected this way are physically unrelated. For each pair a separation of components (\(\Delta \theta_0\)) and the vector of proper motion difference (\(\Delta \mu_0\)) were calculated. We estimated the expected number of unrelated pairs (\(N_{\text{UP}}\)) with angular separations (\(\Delta \theta\)) greater than \(\Delta \theta_0\), the vector proper motion difference (\(\Delta \mu\)) smaller than \(\Delta \mu_0\), and found among stars with proper motions larger than for a given pair (\(\mu_0\)) based on the following equation:

\[
N_{\text{UP}} (\Delta \theta_0, \Delta \mu_0, n(\mu > \mu_0)) = N (\Delta \theta < \Delta \theta_0, n(\mu > \mu_0)) \cdot p (\mu > \mu_0) \tag{3.8}
\]

where \(n(\mu > \mu_0)\) is the number of stars with proper motion larger than \(\mu_0\) in the LMC fields (Eq. 3.5) or in the SMC fields (Eq. 3.6), \(N (\Delta \theta < \Delta \theta_0, n)\) denotes the number of the unrelated pairs with separations smaller than \(\theta_0\) found among \(n\) stars, and \(p (\mu > \mu_0)\) is the probability that the unrelated pair has the proper motion difference smaller than \(\mu_0\).

To evaluate \(N (\Delta \theta < \Delta \theta_0, n)\), we plot the cumulative number of pairs with separation greater than \(\Delta \theta\) in the LMC fields. It is shown in Fig. 3.11. The CPM are expected to have small values of \(\Delta \theta\). The excess of pairs with \(\Delta \theta \lesssim 20''\) is easily seen on the plot. If the stars are distributed randomly on the sky, then the number of stars which lie in the circle of the \(\Delta \theta_0\) radius increases as \(\pi \Delta \theta_0^2\). Thus, we expect \(N (\Delta \theta < \Delta \theta_0, n)\) increases as a square of \(\Delta \theta_0\). Based on over 680,000 pairs with separations between 200'' and 1500'', we found:

\[
N (\Delta \theta < \Delta \theta_0) = 0.31 \Delta \theta_0^2 \tag{3.9}
\]

This relation is shown by the gray line in Fig. 3.11. The slope of the observational data is close to the expected one in the range where the fit was performed. From the fit it can be
estimated that the separation within which one unrelated pair is expected is as small as $1''8$. The value of $N(\Delta \theta < \Delta \theta_0, n)$ increases as a square of the number of stars that were used to found the value of this parameter ($n$). The above relation was found for $n = 10,405$ which gives the scaling factor:

$$N(\Delta \theta < \Delta \theta_0, n) = 2.9 \cdot 10^{-9} n^2 \Delta \theta_0^2$$ (3.10)

The probability $p(\mu > \mu_0)$ heavily depends on the distribution of proper motions in the given population. We plot the proper motion vector-point diagram in $(\mu_\alpha*, \mu_\delta)$ coordinates for the stars in the LMC and SMC fields in Fig. 3.12. As can be seen, both distributions are significantly different. None of them could be well approximated with the bivariate Gaussian distribution for $\mu > \mu_{\text{lim}}$, as they are not symmetric with respect to the solar anti-apex shown in each panel by an arrow. To estimate $p(\mu > \mu_0)$, we used the cumulative distribution function of $\Delta \mu$ for pairs of stars with $\mu > 200''$, of which the great majority.

Figure 3.11: Cumulative distribution of the angular separations between pairs of stars in the LMC fields with $\mu > 30 \text{ mas/yr}$ and $\Delta \theta < 1500''$. Gray line shows the empirical relation for unrelated pairs. Its thick solid part corresponds to the region where it was fitted while dashed part is an extrapolation.
should be unrelated. It is shown in Fig. 3.13 with solid line. As a check, we also plotted with dashed line the same relation for pairs with \( \mu < 7'' \) which should be mostly CPM binaries.

The dashed line increases much faster than the solid one. The cumulative distribution function of \( \Delta \mu \) for the SMC fields overlays with the one for the LMC fields and is not shown for clarity. The only difference was seen at \( \Delta \mu \approx 65 \) mas/yr where we see a slight change of the slope. It is caused by the pairs of stars with proper motions on the opposite sides of the solar apex direction, i.e., \((\mu_\alpha, \mu_\delta) = (0, -30)\) and \((10, 30)\) for the LMC fields.

The values of \( N_{\text{UP}}(\Delta \theta_0, \Delta \mu_0, n(\mu > \mu_0)) \) were calculated for all the pairs with \( \Delta \theta < 1500'' \) and \( \Delta \mu < 30 \) mas/yr. The \( n(\mu > \mu_0) \) was calculated based on Eqs. 3.5 and 3.6. The result was substituted to Eq. 3.10. The final result was calculated according to Eq. 3.8 with \( p(\mu > \mu_0) \) empirically estimated using results presented in Fig. 3.13. The pairs of stars with \( N_{\text{UP}}(\Delta \theta_0, \Delta \mu_0, n(\mu > \mu_0)) < 0.75 \) were recognized as CPM binaries. The smallest values of \( N_{\text{UP}} \) found were below \( 10^{-7} \). This resulted in 316 binaries in the LMC fields and 214 binaries in the SMC fields. For these pairs the sum of \( N_{\text{UP}}(\Delta \theta_0, \Delta \mu_0, n(\mu > \mu_0)) \) is 53.8, what is expected contamination of unrelated pairs in the sample.

The prepared list of the CPM binaries was verified using the RPM diagram. Chanamé & Gould (2004) noted that both components of the CPM binary should lie on the line
parallel to the MS, as they are coeval objects, belonging to the same population. Each CPM binary was flagged as either consistent, inconsistent, or questionable. For 16 pairs in which at least one of the components was a WD could not be verified. There were 52 further pairs for which verification was not possible because one of the components had not color measured. For 59 pairs the verification was not possible because the stars are close to each other on the sky and it was hard to measure the brightness of components. Among the verified pairs there were 298 (77%) marked as consistent, 64 (16%) as inconsistent, and 27 (6.9%) as questionable.5

Figure 3.14 presents the distribution of disk CPM binaries confirmed on the RPM diagram. The logarithm of the number of binaries per bin is shown as a function of logarithm of the angular separation. If we assume that the number of CPM binaries scales as \((\Delta \theta)^{-\alpha}\), then the maximum likelihood estimator results in \(\alpha = 1.50 \pm 0.04\). This is

5These number slightly differ from the ones presented by Poleski et al. (2012) because here we assumed that pairs of stars lying very close to each other in the RPM diagram are verified as consistent.
smaller than 1.67 ± 0.07 found by Chanamé & Gould (2004). The discrepancy might be caused by the stellar streams which may be more pronounced in our data than in the whole-sky sample analyzed by Chanamé & Gould (2004). Median parallax of our CPM binaries is 4.4 mas, what translates to the typical distance almost four times larger than in the sample analyzed by Chanamé & Gould (2004). Our sample seems complete for separations between 2″ and ≈ 1000″, as only the point with log(Δθ ″) ≈ 0.2 deviates from the fitted relation.

Yoo et al. (2004) used the distribution of halo CPM binaries to verify if the dark matter is composed of objects with masses above 43 $M_{\odot}$ (see also Quinn et al., 2009). This limit is significantly larger than the one probed by the microlensing surveys (Wyrzykowski et al., 2011b). Such an investigation is not possible using our sample of confirmed halo binaries because of its small number.

We found one CPM binary containing a WD and a halo star: LMC155.1.4867 and LMC155.1.5999. A similar pair was observed by Monteiro et al. (2006) in order to estimate the age of the red subdwarf. We note that LMC155.1.4867-LMC155.1.5999 pair is less affected by blending than the one analyzed by Monteiro et al. (2006). Also one pair of two WDs was found with a separation of 92″3 (LMC102.7.22769-LMC102.7.22886). There are

Figure 3.14: Distribution of angular separations of the RPM-confirmed disk CPM binaries. The error bars were calculated assuming Poisson statistics.
around 50 such pairs known (Scholz et al., 2002; Andrews et al., 2012), which can be used to test stellar evolution theories. We have also found another fourteen CPM binaries with a WD companions. Such systems can be used to investigate WD initial-final mass relation\textsuperscript{6} (Catalán et al., 2008; Zhao et al., 2012). Another interesting pair, which was confirmed in the RPM diagram, LMC107.2.14205-LMC107.3.195 has very large $\Delta \theta = 593''$ and $\Delta \mu$ of only 0.82 mas/yr. The parallaxes of components are 9.2 ± 1.6 mas and 9.4 ± 1.6 mas. Using these values, we can estimate the projected physical separation as high as 0.31 pc. In the remarks to the catalog, we identified seven triple stellar systems. In each case the separation of two components is significantly smaller than the separation of the third of them.

Our catalog of stellar proper motions can be searched for CPM binaries with a lower limit of $\mu_{\text{lim}}$. The number of candidate CPM binaries, which passed the selection based on the expected number of unrelated pairs, scales as $\mu_{\text{lim}}^{-2.5}$. We can estimate that there will be around 1 460 and 3 000 such systems for $\mu_{\text{lim}}$ of 20 mas/yr and 15 mas/yr, respectively.

3.3.6 Globular cluster 47 Tuc

3.3.6.1 Mean relative proper motion

In order to measure the relative proper motion of globular cluster 47 Tuc and the SMC we selected two subframes of the SMC136 field. We chose this field because it hosts similar number of SMC and 47 Tuc stars and the surface density of stars from a given environment is almost constant. The surface density of 47 Tuc stars in SMC140 rapidly changes with the distance from the cluster center. In order to properly measure the relative proper motion in the SMC140 field, we should fit the coordinate grids to stars either from the SMC or cluster. Currently used solution (grids fitted to all the bright stars and correct at the end) might influenced the proper motion zero point in different parts of the subfields.

We selected two subframes of the SMC136 field with similar number of the 47 Tuc and SMC stars. Stars brighter than 17.7 mag in the $I$ band were taken into account. The average values of the proper motions for stars in both environments were calculated iteratively. In each iteration the uncertainties of the proper motions were square added to the uncertainty of the mean value for given environment and 2.5$\sigma$ clipping was performed or results. The proper motions in one of the subframes are shown in Fig. 3.15. Final

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\textsuperscript{6}Initial-final mass relation gives the mass of the WD as a function of the mass of the MS progenitor.
3.3 Catalog of stellar proper motions

Figure 3.15: Proper motion of 47 Tuc relative to SMC. Gray dots mark the proper motions of the 47 Tuc stars. Black dots represent the proper motions of the SMC stars. Crosses indicate stars not assigned to the SMC or 47 Tuc. The arrow points from the mean proper motion of the SMC to the mean motion of 47 Tuc.

The sample contained 112 SMC stars and 113 belonging to 47 Tuc. The measured proper motion difference of 47 Tuc and the SMC is given in Tab. 3.2 together with the literature measurements. In four cases the absolute proper motion was reported. We transformed these to the relative values by assuming the absolute proper motion of the SMC as reported by Platek et al. (2008), i.e., $\mu_{\text{SMC,} \alpha} = 0.754 \pm 0.061$ mas/yr and $\mu_{\text{SMC,} \delta} = -1.252 \pm 0.058$ mas/yr. The only significantly more accurate measurement than ours is the one based on HST data by Anderson & King (2003). These authors reported not only the mean cluster proper motion but also measured the rotational proper motion with an accuracy of 0.055 mas/yr. The OGLE-III data does not allow measuring mean proper motions with such a good accuracy.

3.3.6.2 Tidal tails

It is well known that some stars leave globular clusters and form tidal tails (e.g., Odenkirchen et al., 2003). The clusters evaporation is governed mainly by the Galactic tidal gravi-
Table 3.2: Relative proper motions of 47 Tuc and SMC

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\mu_\alpha$ [mas/yr]</th>
<th>$\mu_\delta$ [mas/yr]</th>
<th>Data source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tucholke (1992)</td>
<td>5.5 ± 2.0</td>
<td>−1.6 ± 2.0</td>
<td>photographic plates 90 yr apart</td>
</tr>
<tr>
<td>Odenkirchen et al. (1997)(^a)</td>
<td>6.2 ± 1.0</td>
<td>−4.05 ± 1.0</td>
<td>Hipparcos catalog</td>
</tr>
<tr>
<td>Freire et al. (2001)(^a)</td>
<td>5.8 ± 1.9</td>
<td>−2.15 ± 0.6</td>
<td>radioastronomy of pulsars</td>
</tr>
<tr>
<td>Freire et al. (2003)(^a)</td>
<td>4.5 ± 0.6</td>
<td>−2.05 ± 0.6</td>
<td>radioastronomy of pulsars</td>
</tr>
<tr>
<td>Anderson &amp; King (2003)</td>
<td>4.716 ± 0.035</td>
<td>−1.357 ± 0.021</td>
<td>HST data</td>
</tr>
<tr>
<td>Girard et al. (2011)(^ab)</td>
<td>6.9 ± 1.0</td>
<td>−2.1 ± 1.0</td>
<td>SPM4 catalog</td>
</tr>
<tr>
<td>this work</td>
<td>4.41 ± 0.67</td>
<td>−1.12 ± 0.55</td>
<td>OGLE-III data</td>
</tr>
</tbody>
</table>

\(^a\)shifted based on the absolute proper motion of the SMC by Platek et al. (2008); \(^b\)-uncertainty anticipated by the authors is given.

The predicted number density of tidal tails stars in the OGLE-III fields is very small because of the SMC stars which significantly contribute to the background. We could not give any firm conclusions about the existence of 47 Tuc tidal tails using filter matching technique applied to the observed CMD (Odenkirchen et al., 2003). We found another method of verifying the Lane et al. (2012) predictions. Instead of searching for statistical overdensities by filter matching, we decided to search for specific stars that have position in the CMD, sky coordinates, proper motion, and radial velocity as predicted by models. For the follow-up spectroscopic observations, we selected the stars which have both the PM consistent with at least one of the models and for which large ratio of empirical CMD density of the 47 Tuc and SMC was found. Because of technical purposes for our planned spectroscopic observations, we constrained to the $I$-band brightness range 15.5–18.5 mag.

The CMD of the selected stars is shown in Fig. 3.16 with black dots. The gray backgrounds present the CMDs of the 47 Tuc (right panel) and SMC (left panel). The blue envelope of

\(^7\)The proper motions predictions are not a part of Lane et al. (2012) paper and were kindly supplied by the authors. At this point the errors in assumed cluster proper motion was found what resulted in publishing erratum to Lane et al. (2012) paper.
the selected stars is determined by the cluster MS turn-off point. The selected stars avoid the RC and red giant (RG) branch of the SMC. Three stars redder than $(V-I) = 1.2$ mag, if confirmed, would be 47 Tuc binaries. Lane et al. (2012) predicted that the tidal tails in the OGLE-III SMC fields would have proper motions relative to the 47 Tuc proper motion between $-1.3$ mas/yr and $3.3$ mas/yr in $\mu_\alpha$. In $\mu_\delta$ corresponding values are $-2.1$ mas/yr and $2.0$ mas/yr. The proper motions of the selected stars differ from the 47 Tuc by up to $3.6$ mas/yr. The presented catalog of proper motions can be used to search for stars stripped from 47 Tuc using other constrains.

3.3.6.3 New variable stars

We have checked the photometric variability of the 47 Tuc stars by analysing discrete Fourier transform periodograms. Except the known variables (Weldrake et al., 2004), stars with questionable variability, and Galactic foreground stars, we have found three previously unnoticed variables which belong to the 47 Tuc. The proper motions of these stars within uncertainties agree with mean proper motion of the cluster. One of the stars is a firm
SX Phe type pulsator, as it shows two distinct periodicities. Another SX Phe candidate shows only one period and thus is less definite. The last star is a red straggler. All three new variables are marked on the CMD shown in Fig. 3.17. Photometric variability of all three stars is confirmed by the OGLE-IV data collected since 2010. The folded light curves and periodograms of SX Phe pulsators are presented in Fig. 3.18, while raw and folded light curve of the red straggler are presented in Fig. 3.19. We suspect that the variability of these stars was not found so far because of relatively low amplitude (SX Phe pulsators) or period close to two days which hampers its determination (red straggler). Basic properties of our variables are presented in Tab. 3.3.

SX Phe type stars are the blue stragglers stars that cross the δ Sct instability strip. The blue straggler stars are products of either mass transfer in binary systems, direct stellar collisions, or binary mergers (e.g., Geller & Mathieu, 2011). Even though the evolutionary status of blue stragglers is still unclear, their photometric properties are well established. These stars are bluer and brighter than the MS turn-off point.
3.3 Catalog of stellar proper motions

SMC140.7.33598

$P = 0.0459585 \text{ d}$

$P = 0.0509668 \text{ d}$

Figure 3.18: Light curves and periodograms of new SX Phe stars. Each left panel contains the light curve folded with the period corresponding to the highest peak in the periodogram shown in right panel. The first row shows the data for SMC140.7.33598. The second row presents data for this stars after prewhitening with the primary period. The last row shows the data for SMC136.3.2508.
The nature of red stragglers (or sub-subgiants) is even less clear. These objects are defined as stars redder than subgiants with the position in the CMD which cannot be explained by a combination of the light from two MS stars. Almost all of the stars found in this part of the CMD are photometrically variable (Albrow et al., 2001).

The star SMC140.7.33598 with $V = 15.015$ mag is the second brightest SX Phe star known in globular clusters, based on the recently presented list of 263 known SX Phe stars in globular clusters (Cohen & Sarajedini, 2012). The star clearly shows two distinct modes, but our data did not allow deriving their frequencies ambiguously due to daily aliases. The stronger mode has the $I$-band amplitude of 0.007 mag and three possible frequencies:

\[ f_a = 21.75875(20) \text{ 1/d (most probable)}, \quad f_b = 22.76145(20) \text{ 1/d}, \text{ or } f_c = 20.75605(20) \text{ 1/d} \]

\(^8\)The uncertainties of frequencies were calculated using method presented by Schwarzenberg-Czerny (1991).
(least probable). After prewhitening, the second frequency is found with a value of either \( f_d = 19.62060(27) \) 1/d or \( f_e = 20.62335(27) \) 1/d (the peak at \( f \approx 18.6 \) 1/d visible in right middle panel of Fig 3.18 was excluded using OGLE-IV data). The \( I \)-band amplitude of the second mode is 0.003 mag. The possible frequencies for each of the modes are daily aliases.

The periods corresponding to these frequencies are slightly larger than one hour and differ by around three minutes. Unambiguous determination of these frequencies requires a few nights of continues monitoring. We note that if the true frequencies are \( f_b \) and \( f_d \), then their ratio is 0.862. This is very close to the value predicted by theoretical models by Bruntt et al. (2001) for fourth and fifth radial overtone modes. If it is not the case, at least one of the modes is non-radial.

For the second SX Phe candidate (SMC136.3.2508) we found only one mode with an \( I \)-band amplitude of 0.010 mag and frequency of 23.4450591(83) or its daily alias. We note that only five SX Phe stars were discovered so far in 47 Tuc (Cohen & Sarajedini, 2012).

The position of the red straggler candidate (SMC140.8.15169) on the CMD is typical for this type of stars. Its photometric period is 2.018223(38) d. The light curve is similar to spotted, rotating variable stars. The red stragglers in 47 Tuc overlap in the CMD RGs from the SMC RGs. However, these stars cannot be SMC members, as they are observed only inside the tidal radius of 47 Tuc and almost all of them show photometric variability not seen in the SMC RGs (Albrow et al., 2001). The fact that the proper motion of SMC140.8.15169 is consistent with the mean proper motion of 47 Tuc gives additional evidence for cluster membership of red stragglers.

### 3.3.7 Absolute proper motions of the Magellanic Clouds

The absolute proper motions of the Magellanic Clouds is of major astrophysical importance. If known accurately, they may be used to constrain the mass enclosed in the Milky Way inside spheres with radii of 50 kpc and 60 kpc. Combined with the radial velocities, they
allow to simulate past and future changes of relative positions of both Clouds and the Milky Way. Such simulations can be compared with the observed star formation history. Even though the absolute motions are important, they are not known accurately. It is still not clear if the Magellanic Clouds are on their first passage to the Galaxy or have made a few orbits since their formation.

Performing high signal-to-noise measurements of the absolute proper motions is hampered not only by the small values of the proper motions but also by the geometric effects. Particularly in the case of the LMC, one has to take into account a handful of effects: the motion of the center of mass, both precession and nutation of the LMC disk, internal rotation which varies with distance from the galaxy center, and different distances to different parts of the galaxy. The impact of all these factors on the proper motion measured in different parts of the sky was presented by van der Marel et al. (2002). The resulting equations are rather complicated and fitting all the necessary parameters is not straightforward and is affected by the small number statistics.

The absolute proper motions of the Magellanic Clouds can be measured using either the background galaxies or quasars. The galaxies used as reference points are problematic for ground-based observations because the apparent centroid may change with changing seeing.

The most precise LMC proper motion published to date are those that based on the HST observations. Kallivayalil et al. (2006) found $\mu_{\alpha*} = 2.03 \pm 0.08$ mas/yr, $\mu_\delta = 0.44 \pm 0.05$ mas/yr while re-analysis by Piatek et al. (2008) resulted in $\mu_{\alpha*} = 1.95 \pm 0.04$ mas/yr, $\mu_\delta = 0.44 \pm 0.04$ mas/yr. We cross-match the list of known quasars behind the LMC (Kozłowski et al., 2012) with our catalog of proper motions and selected objects brighter than $I = 18$ mag. The nine selected quasars resulted in the LMC proper motion of $\mu_{\alpha*} = 2.61 \pm 0.16$ mas/yr, $\mu_\delta = 0.46 \pm 0.26$ mas/yr. The difference between our result and the HST ones in $\mu_{\alpha*}$ is significant. Possible cause of this may be small sample used by us. We note that if more bright and isolated quasars are found towards the LMC and SMC, one can calculate proper motions of these galaxies using the catalog of proper motions presented here.
3.3.8 Proper motions of variable stars

The proper motions of variable stars are presented separately. First, because in the dense stellar field reliable calculation of their proper motions is harder than for the non-variable objects. Second, because they can be used in dedicated research. As an example we note RR Lyr variables, for which absolute brightness can be found based on their photometric properties. Martin & Morrison (1998) and Kinman et al. (2007) used these variables to analyze their distribution and kinematics.

We note that there are three astrophysical situations in which the measured proper motion of a variable stars in the LMC or SMC field can be significant:

1. the variable star is located in the foreground of the Magellanic Clouds,

2. the variable star is located in the Magellanic Clouds and blended with the foreground object,

3. Magellanic Cloud variable has an exceptionally high tangential velocity and it is possibly a runaway object.

The first possibility can be verified using the CMD or the absolute brightness derived from the photometric properties. The second one can be judged by the imaging with high spatial resolution. The last possibility is quite unlikely, but if such a star is found, it could be used to investigate the dynamics of the Galaxy and Magellanic Clouds system.

We visually inspected all the objects from the OGLE-III Catalog of Variable Stars for which models with significant proper motions were fitted. Special attention was paid to check if the change of the stellar position is not caused by the blending coupled with stellar variability. In total, 237 variables with significant proper motion were found. They are presented in a separate file in our catalog. Among these stars, there are 162 eclipsing binaries and 25 RR Lyr variables. There are two CPM binaries containing variable stars. They are indicated in the remarks file.

We found two published papers in which our proper motions of variable stars contradict the conclusions presented by authors. In the first of them, Marquette et al. (2008) investigated a star which in their opinion was “a peculiar Cepheid-like star.” We found proper motion of that star of 7.4 ± 1.2 mas/yr, what clearly shows it does not belong to the SMC. This star has a light curve typical for spotted stars.
The second research in which our proper motions of variable stars change the conclusions was the one by Sabogal et al. (2005). These authors claimed that in the LMC blue variables separate into two groups in the CMD. The first group contains the stars with \((B - V)\) color mostly between \(-0.3\) mag and 0.0 mag. The second group has 0.4 mag < \((B - V)\) < 0.6 mag and it was not seen in the SMC (Mennickent et al., 2002). We have cross-match the stars from the two groups with our catalog of stellar proper motions. We found that the second group contains the stars which do not belong to the LMC but are foreground Galactic stars. In the second group 87.5% of stars had \(\mu > 4\) mas/yr while in the first this number was 0.5%. In our opinion, the photometric variability found by Sabogal et al. (2005) was caused by the change of stellar centroid, which affects the DIA photometry (Alcock et al., 2001; Eyer & Woźniak, 2001). Our finding is consistent with the color-color diagram shown by Sabogal et al. (2005). In this diagram the stars from the second group follow the MS track of spectral types later than F5. We note that the origin of this group of stars with 0.4 mag < \((B - V)\) < 0.6 mag was not known so far (Paul et al., 2012).

One further application of our catalog of proper motions in the variable stars research is connected to the predicted pulsations of low-mass MS stars. Recently, Rodríguez-López et al. (2012) presented pulsation models of MS stars with masses between 0.1 \(M_\odot\) and 0.5 \(M_\odot\). They found unstable modes but no pulsations with predicted properties were reported. Using our proper motions catalog a statistically significant sample of nearby MS stars can be selected. The Rodríguez-López et al. (2012) hypothesis can be verified, because even if the pulsation periods are too short to be detected using our data (e.g., around half an hour), one can select stars with dispersion of brightness measurements larger than expected from the photometric noise. A few hours of dedicated observations for each of such selected candidates should reveal pulsations if they are present.

### 3.3.9 Cepheid instability strip

One more application of our catalog is the search for stars lying inside the classical Cepheids instability strip which are not pulsating. To bound the Cepheid instability strip, we used the catalog of classical Cepheids in the LMC published by Soszyński et al. (2008). Obviously, the main contaminants in the part of the CMD where Cepheids reside are Galactic foreground stars. Our catalog can be used to remove the great majority of these contami-
Figure 3.20: Selection of the CMD area where most of the Cepheids reside. Left panel shows the $I$-band brightness vs. the $I$-band amplitude for fundamental mode Cepheids. Right panel presents the same stars on the CMD diagram. Gray lines constrain the part of the CMD from which constant stars were further analyzed.

Figure 3.21: Sky-positions of the LMC non-pulsating stars which are located in the part of the CMD populated by Cepheids. The background image originates from the ASAS survey (Pojmański, 1997).
nants.

In order to select stars that lie in the instability strip, we used the distributions of classical Cepheids presented in Fig. 3.20. The left panel presents the mean $I$-band brightness vs. the $I$-band amplitude for Cepheids. We intended to restrict to the range of $I$-band brightness in which Cepheids are effectively found and high accuracy proper motions are available (Fig. 3.8). We selected the range between 13.5 mag and 15.5 mag (shown by the horizontal gray lines of both panels of Fig. 3.20). Most of the Cepheids in this brightness range have amplitudes between 0.3 mag and 0.6 mag. The OGLE-III catalog of the LMC Cepheids is practically not limited by the photometric accuracy and the catalog completeness is very high. Additional constraints were selected using the CMD (right panel of Fig. 3.20). They are shown by the inclined lines. In total, 14,840 non-variable stars in the part of the CMD restricted by the four lines were selected. The Galactic foreground stars were removed by imposing constrains on the proper motion: it had to be smaller than 2 mas/yr and consistent with zero within $2\sigma$ limit. This reduced the number of stars to 1,361. The positions of these stars are shown in Fig. 3.21. They clump in the LMC bar. It is a strong suggestion that most of these stars are the LMC objects.

One can speculate that these stars are either nearby MS stars or WDs with very small proper motions or their photometry is affected by blending, etc. The second suggestion can be verified in some cases using observations with higher spatial resolution, e.g., archival HST images. The hypothesis that these objects are nearby can be verified using already performed Strömgren photometry of selected LMC fields. These two tests will allow one to map the Cepheid instability strip or find objects within the strip which are not pulsating.
Chapter 4

Analysis of selected fields toward the Galactic bulge

In this chapter, we present our investigation of proper motions of the two arms of the X-shaped structures. First, we select the stars that belong to both arms. Next, we present the image reduction procedure and the method of proper motion calculation. The proper motions statistics for the two arms are derived and compared with other results in the last section.

4.1 Double red clump in selected fields

The goal of this section is to calculate probabilities that a given star belongs to the brighter or fainter part of the X-shaped structure. We describe the fields chosen, the dereddening process, selection of stars from the CMD, construction of the luminosity function (LF), the function which is fitted to the LF, the fitting results, and final probabilities.

The CMDs in the fields towards the bulge show both the MS stars brighter than the turn-off point and RGs. The MS stars belong to the disk, while the RGs are the members of the bulge (see, e.g., Kuijken & Rich, 2002). The double RC is best seen in the LF of the bulge RGs. However, the part of the CMD where bulge giants reside contains, except the single or double RC, also other features. Gallart (1998) discussed the RC, red giant branch bump (RGBB), and asymptotic giant branch bump (AGBB). The first evidence for existence of the RGBB and AGBB in the Galactic bulge was reported by Nataf et al. (2011b). The AGBB is a very weak (number ratio to RC of $\approx 1.5\%$) structure brighter
by \approx 1.1 \text{ mag} \text{ than the RC}. On the other hand, the RGBB is fainter than the RC by 0.737 \text{ mag} (i.e., it is similar to the magnitude difference of the two RCs) with number ratio of 20.1\% (Nataf et al., 2011a). The existence of RGBB in the bulge was also confirmed by Gonzalez et al. (2011) and Clarkson et al. (2011). The RGBB was taken into account with parameters derived by Nataf et al. (2011a) or fitted. For simplicity, we assumed the dispersion of RGBB brightness to be the same as the RC one. In order to avoid confusion, both the brighter RC and the brighter RGBB are called “the brighter arm” (of the X-shaped structure), while the fainter RC and the fainter RGBB are called “the fainter arm.”

The double RC is visible in the fields South of \( b \approx -5^\circ \) and North of \( b \approx 5^\circ \). During the OGLE-III survey, the bulge fields were observed with various frequency and time span. Among the fields with double RCs, we chose the ones with the best time coverage for measuring proper motions. These four fields are presented in Tab. 2.1. None of the fields North of the Galactic disk with the double RC has good observational coverage for the proper motion calculations.

![Figure 4.1: Color-magnitude diagram for the stars in the subfield BLG176.5. The left panel shows the observed CMD, while the right one presents the dereddened one. Stars lying left of the gray line were used to construct the L.F.](image)
To deredden the brightness of each star, we used the extinction maps presented by Nataf et al. (2012). These maps are based on the RC position on the CMDs. In order to estimate the reddening between the grid points, we performed the Delaunay triangulation\(^1\) of the grid points, which were represented on the \((\alpha \cos \delta, \delta)\) plane. The reddening towards each star was a linear interpolation of the three grid points that define the triangle in which the given star lies. The same procedure was used to evaluate extinction in the \(I\)-band. The reddening and extinction were subtracted from the observed \((V-I)\) and \(I\), respectively, which resulted in the reddening corrected values \(((V-I)_0\) and \(I_0)\). Fig. 4.1 presents as an example the CMD in the BLG176.5 subfield. We note that this procedure did not correctly dereddened the disk stars which are on average closer than the gas which reddens the light we observe. It is most easily seen for the MS stars. This disadvantage is not important to us because the MS stars bluer than \((V-I)_0 = 0.6\) mag were not incorporated in the constructed LFs.

The LF of the bulge RGs is constructed separately for each analyzed field using the \(I_0\) vs. \((V-I)_0\) CMD of that field. We restricted our LF to the part of the CMD marked in the right panel of Fig. 4.1, which shows the example subfield BLG176.5. The vertical cut at \((V-I)_0 = 0.6\) mag removed the disk MS stars. The inclined line for stars brighter than \(I_0 = 14\) mag removed the contamination of disk RC stars. The contamination of the disk stars redder than \((V-I)_0 = 0.6\) mag is not removed.

The next part of the analysis was a construction of the LF. We applied the kernel density estimation method, which is a form of generalized histograms. Instead of binning the data, as it is done during the histogram construction, every value is replaced by a kernel function. The kernel functions of all the nearby points are summed to obtain the value of the LF for given \(I_0\). Mathematically,

\[
\text{LF} (I_0) = \frac{1}{h} \sum_i K \left( \frac{I_0 - I_{0,i}}{h} \right) \tag{4.1}
\]

where \(h\) is a bandwidth that defines the amount of smoothing used, \(K(u)\) is the kernel, and

\(^1\)Delaunay triangulation gives for a given set of points the subdivision of the plane, on which they lie, into non-overlapping triangles. The triangle edges are the points of the given set. No of the triangle edges lies inside the circumcircle of any triangle.
\(I_{0,i}\) is the dereddened brightness of individual stars. We choose the Epanechnikov kernel:

\[
K(u) = \frac{3}{4} (1 - u^2)
\]

defined for \(-1 \leq u \leq 1\). In that case the summation in Eq. 4.1 is done for dereddened magnitudes in the range \(I_0 - h \leq I_{0,i} \leq I_0 + h\). We choose \(h = 0.06\) mag and 0.03 mag sampling.

To find the probability that a given star belongs to either brighter or fainter RC, we fitted the obtained LFs with the following model from Nataf et al. (2010, 2011b). We fitted the function which contained five components:

\[
\text{LF} (I_0) = n_{\text{RG}} (I_0) + n_{\text{bRC}} (I_0) + n_{\text{bRGBB}} (I_0) + n_{\text{fRC}} (I_0) + n_{\text{fRGBB}} (I_0)
\]

The \(n_{\text{RG}} (I_0)\) denotes the RG population as well as the contamination by disk and halo objects. We assumed it to be an exponential function, while four other components are Gaussian functions. The bright arm of the X-shaped structure contributes through its RC \(n_{\text{bRC}} (I_0)\) and RGBB \(n_{\text{bRGBB}} (I_0)\). Similarly, the contribution of the fainter arm is described by \(n_{\text{fRC}} (I_0)\) and \(n_{\text{fRGBB}} (I_0)\). The components are defined by the following equations:

\[
n_{\text{RG}} (I_0) = A \exp (B (I_0 - I_{0,\text{bRC}}))
\]

\[
n_{\text{bRC}} (I_0) = \frac{N_{\text{bRC}}}{\sqrt{2\pi}\sigma_{\text{bRC}}} \exp \left(\frac{-(I_0 - I_{0,\text{bRC}})^2}{2\sigma_{\text{bRC}}^2}\right)
\]

\[
n_{\text{bRGBB}} (I_0) = \frac{N_{\text{bRGBB}}}{\sqrt{2\pi}\sigma_{\text{bRGBB}}} \exp \left(\frac{-(I_0 - I_{0,\text{bRGBB}} - \delta I_{0,\text{bRGBB}})^2}{2\sigma_{\text{bRGBB}}^2}\right)
\]

\[
n_{\text{fRC}} (I_0) = \frac{N_{\text{fRC}}}{\sqrt{2\pi}\sigma_{\text{fRC}}} \exp \left(\frac{-(I_0 - I_{0,\text{fRC}})^2}{2\sigma_{\text{fRC}}^2}\right)
\]

\[
n_{\text{fRGBB}} (I_0) = \frac{N_{\text{fRGBB}}}{\sqrt{2\pi}\sigma_{\text{fRGBB}}} \exp \left(\frac{-(I_0 - I_{0,\text{fRGBB}} - \delta I_{0,\text{fRGBB}})^2}{2\sigma_{\text{fRGBB}}^2}\right)
\]

Where \(A\) is the number of the RG stars at the RC brightness, \(B\) defines the slope of the RG stars LF, \(\sigma_{i,j}\) are the dispersions of brighter (subscript \(i = b\)) and fainter (subscript \(i = f\)) of the RC and the RGBB (denoted by \(J\) subscript), \(N_{i,j}\) are number of stars in given populations with the same subscript convention, \(I_{0,\text{bRC}}\) and \(I_{0,\text{fRC}}\) are dereddened magnitudes of the brighter and fainter RC, \(\delta I_{0,\text{bRGBB}}\) and \(\delta I_{0,\text{fRGBB}}\) are the brightness
4.1 Double red clump in selected fields

differences between RC and RGBB in the brighter and fainter arms.

The final model given by the above equations contains 14 parameters. The number of the input data-points in each of the LFs was around 100. In order to fit the model correctly, we reduced the number of free parameters to maximum of nine by using the following constrains:

$$\sigma_{\text{RGBB}} = \sigma_{\text{RC}}$$  \hspace{1cm} (4.9)

$$\frac{N_{\text{RGBB}}}{N_{\text{RC}}} = 0.201$$  \hspace{1cm} (4.10)

$$\delta I_{0,\text{RGBB}} = 0.737 \text{ mag}$$  \hspace{1cm} (4.11)

for both the brighter and the fainter arm. The second and third constrains are the values found by Nataf et al. (2011a). The bright ($I_{0,\text{min}}$) and faint ($I_{0,\text{max}}$) limits of the part of the LF which were fitted were chosen by trial and error, so that the other populations do not contribute to the LF and low number counts in the bright end do not affect our results. In the field BLG134 the fit was performed without contribution of the RGBB stars from the fainter arm. This assumption gave much better fits than the ones with $N_{\text{RGBB}}$ as a free parameter or $N_{\text{RGBB}}/N_{\text{RC}} = 0.201$. Because of similar reasons, $N_{\text{RGBB}}/N_{\text{RC}}$ was a free parameter in BLG176. The fitting was performed using the Levenberg-Marquardt algorithm implemented by Press et al. (1992).

Table 4.1: Fitted Luminosity Functions parameters

<table>
<thead>
<tr>
<th></th>
<th>BLG134</th>
<th>BLG160</th>
<th>BLG167</th>
<th>BLG176</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>288 ± 46</td>
<td>178 ± 35</td>
<td>261 ± 37</td>
<td>223 ± 37</td>
</tr>
<tr>
<td>$B$ [1/mag]</td>
<td>0.671 ± 0.011</td>
<td>0.651 ± 0.015</td>
<td>0.612 ± 0.011</td>
<td>0.646 ± 0.013</td>
</tr>
<tr>
<td>$I_{0,\text{RC}}$ [mag]</td>
<td>14.290 ± 0.032</td>
<td>14.261 ± 0.040</td>
<td>14.294 ± 0.043</td>
<td>14.238 ± 0.039</td>
</tr>
<tr>
<td>$N_{\text{RC}}$</td>
<td>112 ± 20</td>
<td>126 ± 17</td>
<td>202 ± 34</td>
<td>170 ± 26</td>
</tr>
<tr>
<td>$\sigma_{\text{RC}}$ [mag]</td>
<td>0.180 ± 0.019</td>
<td>0.254 ± 0.025</td>
<td>0.238 ± 0.023</td>
<td>0.230 ± 0.021</td>
</tr>
<tr>
<td>$\delta I_{0,\text{RC}}$ [mag]</td>
<td>0.737</td>
<td>0.737</td>
<td>0.737</td>
<td>0.737</td>
</tr>
<tr>
<td>$N_{\text{RGBB}}/N_{\text{RC}}$</td>
<td>0.201</td>
<td>0.201</td>
<td>0.201</td>
<td>0.201</td>
</tr>
<tr>
<td>$\sigma_{\text{RGBB}}/\sigma_{\text{RC}}$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$I_{0,\text{RC}}$ [mag]</td>
<td>14.650 ± 0.016</td>
<td>14.696 ± 0.024</td>
<td>14.661 ± 0.030</td>
<td>14.628 ± 0.027</td>
</tr>
<tr>
<td>$N_{\text{RC}}$ [mag]</td>
<td>186 ± 19</td>
<td>69 ± 17</td>
<td>90 ± 33</td>
<td>102 ± 26</td>
</tr>
<tr>
<td>$\sigma_{\text{RC}}$ [mag]</td>
<td>0.160 ± 0.010</td>
<td>0.162 ± 0.020</td>
<td>0.158 ± 0.020</td>
<td>0.163 ± 0.018</td>
</tr>
<tr>
<td>$\delta I_{0,\text{RGBB}}$</td>
<td>0.737</td>
<td>0.737</td>
<td>0.737</td>
<td>0.737</td>
</tr>
<tr>
<td>$N_{\text{RGBB}}/N_{\text{RC}}$</td>
<td>0.000</td>
<td>0.201</td>
<td>0.201</td>
<td>0.088 ± 0.063</td>
</tr>
<tr>
<td>$\sigma_{\text{RGBB}}/\sigma_{\text{RC}}$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$I_{0,\text{min}}$ [mag]</td>
<td>13.10</td>
<td>12.76</td>
<td>13.00</td>
<td>12.67</td>
</tr>
<tr>
<td>$I_{0,\text{max}}$ [mag]</td>
<td>15.60</td>
<td>15.40</td>
<td>15.80</td>
<td>15.70</td>
</tr>
</tbody>
</table>

Values given without uncertainties were fixed during the model fitting.

The results of fitting are presented in Tab. 4.1 and Fig. 4.2, where we plot $n_{\text{RG}} (I_0) +$
Figure 4.2: Luminosity functions and fitted models. Gray dots are measured values. Solid black lines show the best model. The dotted line presents the contribution of RGs and the brighter arm while the dashed line presents the contribution of RGs and the fainter arm.
\( n_{bRC} (I_0) + n_{bRGBB} (I_0) \) and \( n_{RG} (I_0) + n_{RC} (I_0) + n_{RGBB} (I_0) \) separately, as well as the full model fitted. The uncertainties were taken from the covariance matrix. To calculate this matrix, we assumed the uncertainties of the LF to be the same as in the Poisson distribution, i.e., \( \sigma_{LF(I_0)} = \sqrt{LF(I_0)} \).

The constructed fits to the LFs were used to estimate the probabilities that star of a given \( I_0 \) belongs to the brighter \( (p_b (I_0)) \) or the fainter \( (p_f (I_0)) \) arm of the X-shaped structure. We define this probabilities as the ratios of the number of stars of given \( I_0 \) in the arm under consideration to all the stars of given \( I_0 \):

\[
p_b (I_0) = \frac{n_{bRC} (I_0) + n_{bRGBB} (I_0)}{n_{RG} (I_0) + n_{bRC} (I_0) + n_{RGBB} (I_0) + n_{RC} (I_0) + n_{RGBB} (I_0)} \tag{4.12}
\]

\[
p_f (I_0) = \frac{n_{RC} (I_0) + n_{RGBB} (I_0)}{n_{RG} (I_0) + n_{bRC} (I_0) + n_{RGBB} (I_0) + n_{RC} (I_0) + n_{RGBB} (I_0)} \tag{4.13}
\]

The plots of \( p_b (I_0) \) and \( p_f (I_0) \) are shown in Fig. 4.3. Except the BLG134 field the maximum values of \( p_b (I_0) \) are between 0.53 and 0.56. The maximum of \( p_f (I_0) \) ranges between 0.34 and 0.40. For the BLG134 field the trend is opposite, i.e., 0.45 for \( p_b (I_0) \) and 0.53 for \( p_f (I_0) \). This finding is consistent with Saito et al. (2011) results. We note that at the brightness at which \( p_f (I_0) \) maximizes the values of \( p_b (I_0) \) are between 0.14 and 0.17 for the fields near \( l \approx 0^\circ \). This significant contribution of the brighter RC is caused by the RGBB stars, thus neglecting them in Eq. 4.3 would result in biased probabilities.

### 4.2 Image reduction

While analyzing the time-series astrometry from the OGLE-III DoPilot reduction, we had to apply corrections to the centroids measured in each image. The accuracy of the results might be slightly worse than the best possible with a given dataset. More severe problems would be encountered if this data analysis method was applied to the Galactic bulge fields. Because of the high stellar density, the standard OGLE-III reduction was performed on 545 × 522 pixel subframes. Such small subframes contain smaller number of stars to which the astrometric grids can be fitted. Another disadvantage is a significant proper motion of every star in the field as the proper motion dispersions in the bulge are between 2 and 3.5 mas/yr and are higher for disk stars (see summary presented by Vieira et al., 2007). If the bulge fields were reduced in small subframes, the proper motion zero
Figure 4.3: Probability that a star with given $I_0$ belongs to the either RC. Dashed line presents $p_b(I_0)$, while full line presents $p_f(I_0)$. 

BLG134 

BLG160 

BLG167 

BLG176
points would have changed on the small angular scales.

These disadvantages forced us to perform the additional reduction of the OGLE-III bulge data. In order to achieve high accuracy, we resign from using the DoPHOT software. We decided to write our own software based on the algorithm presented by Anderson & King (2000). They introduced an effective PSF method, which turned out to be very accurate and efficient in determining the stellar centroids in the HST images. In the effective approach, the PSF represents the fraction of stellar light that falls on the pixel as a function of the offset of the pixel from the stellar centroid. In the instrumental approach, the PSF represents the fraction of stellar light per unit area that falls on the point of the detector as a function of the offset of the point from the stellar centroid. Thus, the value of the effective PSF is the integral of the product of the instrumental PSF and the pixel sensitivity profile over the area of one pixel. As noted by Anderson & King (2000), the effective PSF is more convenient for doing astrometry, as it contains the information that we can gain from analysing the pixel values of the bright stars. It also contains all the only information that we need to find the stellar centroids and fluxes. Instrumental PSF approach is used, e.g., in the DAOPHOT software. The integration of the PSF over a pixel is performed and the result is used in fitting the centroid and the flux.

The effective PSF approach makes the biggest profit when space-based images are analyzed. In that case consecutive images are taken in almost the same conditions, while the ground-based images are affected by atmosphere changes. Still, this method allows accurate modeling of the PSF which is a key point in deriving the stellar centroids with high accuracy. Since Anderson & King (2000) pointed out the disadvantages of the existing software, which is intended in performing good photometry and not necessarily good astrometry, no software performing high accuracy astrometry on the ground-based images was made publicly available. There are such programs that implement effective PSF method on the HST images, but they use a pre-defined templates of the HST PSF profiles. The effective PSF method was already successfully applied to the ground-based images from the camera similar to the OGLE-III one, namely the Wide Field Imager at ESO 2.2 m ESO/MPI telescope (Anderson et al., 2006; Yadav et al., 2008; Bellini et al., 2009).

Below we describe our own software ASTROWARS\(^2\) and its performance. The detailed

\(^2\)ASTROWARS is the software developed at the University of Warsaw in order to perform high-accuracy ASTROMetry by analogy with the DAOPHOT which was developed at the Dominion Astrophysical Observatory in order to perform high-accuracy PHOTometry.
description of the algorithm used is presented in the Appendix A. It was already used to investigate the proper motions in the vicinity of the South Ecliptic Pole (Soszyński et al., 2012).

The model of the PSF in AstroWARS is kept as a tabulated function of the offsets from the centroid position in two directions of the detector coordinates. The offset between the the stellar centroid and the point in which PSF has to be evaluated can be any real number. In order to evaluate PSF accurately, AstroWARS tabulates the PSF on the grid with a resolution element one quarter of the pixel, i.e., it super-samples the pixel by a factor of four. The values between the grid points are found by the bi-cubic spline interpolation.\footnote{Bi-cubic spline is a type of interpolation defined on a two-dimensional regular grid of points. It conserves the function values and derivatives in the grid points and result is a continuous function with continuous first derivatives. It is an extension of the cubic spline interpolation to functions defined on two dimensions.}

The PSF changes with the position on the detector. Hence, if a single PSF model is used to process the whole image, the resulting centroids have systematic errors. We followed Anderson et al. (2006) with the treatment of the PSF changes with the detector position. AstroWARS divides the detector into subframes and has a separate PSF model for every subframe. We used a grid of $2 \times 4$ subframes (i.e., each of them was $1024 \times 1024$ pixels) as shown in Fig. 4.4; however, the software has a capability of using any grid of subframes. Subframe borders are shown in Fig. 4.4 with dotted lines and position to which they correspond are marked by letters $a$, $b$, $c$, etc. To evaluate the PSF in a particular detector location, the linear interpolation of the four nearest subframe PSFs is performed. An example position is marked by a "$\star$" sign and the arrows connect the four positions to which PSFs subframe PSF correspond with the point where the PSF is evaluated. To derive the PSF in each subframe AstroWARS uses the bright stars that are in the given subframe.

The approach of selecting points to which the subframe PSF corresponds to (i.e., locations of the circles in Fig. 4.4), has an essential disadvantage: the mean position of the stars used to derive PSF is typically close to the subframe center, but we assume the PSF corresponds to the point on the edge of the subframe. This affect most the corner points (i.e., $a$, $b$, $g$ and $h$). If there is a bright star in the position marked in Fig. 4.4, it is used to derive subregion PSF marked by $b$, but it is closest to the subregion PSF marked by $d$ and this PSF gives biggest contribution during interpolation. This problem is partly overcome because PSF is derived iteratively, and in each iteration from each PSF star we subtracted the model
4.2 Image reduction

Figure 4.4: Schematic explanation of the PSF interpolation. Numbers give the pixel coordinates. Solid lines present the edges of the image frame. Dotted lines divide the image frame into subframes. In every subframe a separate model of the PSF is found. It is assumed that it corresponds to the point which is marked by a circled letter. The arrows show how the interpolation is performed for an example point on the frame (marked by a star). This follows the method presented by Anderson et al. (2006).

resulting from the previous iteration, which evaluation takes into account all four closest subregion PSFs. Even though this disadvantage of the approach presented in Fig. 4.4 is known, no better approach was found.

Example plot of the PSF profiles returned by AstroWars is presented in Fig. 4.5. The complex dependence of the profile on the image position is easily recognized. The values of the PSF central pixel are significantly smaller at the image edges than at the center, due to the comatic aberration.

Anderson & King (2000) noted that if the PSF model is not good enough, the measured centroids have systematic errors with respect to the pixel boundaries. They called it the pixel-phase error. We compared the influence of this error on the centroids measured with
Figure 4.5: Example plot of PSF profiles in a single frame. Each profile presents central $10 \times 10$ pixels part of the profile. Black lines separate eight subframes of the camera. The FWHM of the PSF varies between 3.9 and 4.3 pixels.
Figure 4.6: Pixel-phase error. The left panel shows the centroids measured using AstroWARS. The right panel presents the centroids measured using img2xyM_WFI for the faint stars only. Each panel presents 10,000 randomly selected centroids, many of which overlap.

our software and with img2xyM_WFI, which was used by Anderson et al. (2006), Yadav et al. (2008), and Bellini et al. (2009) to analyze the ground-based images taken with camera similar to the OGLE-III one. The fractional parts the X and Y coordinates of the stellar centroids measured using both programs on the same image\(^4\) are presented in Fig. 4.6. The left panel presents the centroids measured using AstroWARS (whole brightness range) while the right panel shows the centroids measured using img2xyM_WFI for the stars with fluxes smaller than 3,300 ADU (faint stars). In the case of AstroWARS significantly larger number of stars have the fractional parts of coordinates between 0.48 and 0.52. The area, which covers 7.8% of the plot, contains 20% of stellar centroids. In the case of the faint stars measured using the img2xyM_WFI the problem is more severe, but the centroids of bright stars are not affected. We counted the centroids with fractional parts of at least one of the coordinates equal to either 0.000, 0.250, 0.333, 0.500, 0.667, or 0.750. It turned out that 64% of the faint stars are in this subset and the selected area covers 1.2% of the plot. Most of the other centroids have fractional parts equal to the ratio of two small integers. As can be seen, both programs suffer from the pixel-phase error but in a different way.

\(^4\)The image and the img2xyM_WFI results were kindly supplied by Dr. Andrea Bellini.
4.3 Calculation of proper motions

In this section, we present the methodology of the proper motions calculation in the bulge fields. We had centroids measured using ASTROWARS, which aims in precise astrometry. Thus, smaller number of corrections had to be applied and this part of the analysis was simpler than in the case of Magellanic Clouds fields.

The very important step of the proper motion calculation is finding appropriate grid transformations. The analyzed images had, in some cases, large relative shifts (up to 150 pixels in one direction) and differed in seeing FWHM up to a factor of 2.5. The grid transformations had to be found many times and using a relatively large number of stars (> 1000). To properly find the grid transformations, we developed our own software. It first cross-matched the two star lists and then iteratively finds the quadratic grid transformation coefficients. We tried using cubic grids but it resulted in unphysically large values of transformation corrections on image edges. The most time consuming part of the calculations is cross-matching between the lists of stars from two images. It is frequently done by constructing a set of triangles from each list and then finding similar triangles. Our software constructs all the triangles from the given set of points with sides larger than 20 pixels and smaller than 500 pixels. The triangles with very small and very large sides are omitted as they carry very little information. Such a restriction on the side lengths significantly reduced the number of constructed triangles which grows as a cubic of the number of points. The triangles were kept in the space defined by Pál & Bakos (2006). This triangle space is continuous function of sides for nonsingular triangles, thus, a small change of side changes the triangle position in the space by a small amount. This space also conserves chirality of the triangle and in typical situation triangles fill the unit circle almost uniformly. When the triangles from the two star lists were constructed, they were cross-matched. In this cross-match two triangles were assign if the first one was the closest to the second one from the first list, as well as the second triangle was the closest one to the first triangle from the second star list (“nearest-to-nearest” algorithm). After cross-matching the triangles the vote table was constructed. Its size was equal to the number of stars in the first star list times the number of stars in the second star list. At the beginning all the cells in the vote table were equal to zero. A cross-match pair of triangles gives three cross-matching pairs of stars. The cells of the vote table which
4.3 Calculation of proper motions

correspond to this three pairs of stars were increased by one. This was repeated for every cross-match pair of triangles. At the end, the cell of the vote table with this highest value corresponds to the pair of stars which was assigned most frequently. Typically, the number of votes in this cell was larger than the number of stars in the input list. The pairs of stars with the number of votes larger than \( \approx 10\% \) of the number of stars in the input lists were used to find grid transformation. Additionally, it was checked if the linear coefficients were consistent with the transformation containing only shift, rotation, and dilation. In all cases, when quantity defined by (Pál & Bakos, 2006, their Eq. 20) was significantly larger than zero, the transformation was spurious.

The above described cross-matching procedure turn out to be very crude and efficient. It was used in all further cross-matches of the star lists.

The first step in deriving the proper motions was building a database of centroids derived by AstroWARS. To do this, we decided to prepare our own database format. It was similar to the one presented by Szymański & Udalski (1993) and since then used by the OGLE group with modifications. We wanted to have a full control on the fitted grids, thus in the database we stored the centroids measured on the raw images as well as grids that transform it to our reference coordinates. The transformations from the pixel coordinates of the OGLE-III images to the equatorial ones was calculated earlier and in order to simplify our analysis, we used these transformations. As reference coordinates in the database, we taken the coordinates (both pixel and equatorial ones) transformed to the coordinates of the image which was taken as the first one to construct the reference image. The database was stored in three files. The first one contains the index of images with basic information, such as a time of observation, exposure time, seeing FWHM, and sky background. For every image also a 24-element table was stored. It defines the quadratic transformation from given image coordinates to the reference image coordinates and backwards. Also the \( \text{rms} \) of each transformation was stored. The second file contains the list of stars with their mean \( I \)-band brightness and its dispersion, \( (V-I) \) color, and reference position. The third file was the largest as it contained individual measurements for every star. The single record contained difference between measured and mean brightness, brightness uncertainty, the difference between the measured position and the reference one (two coordinates), and the uncertainty of the measured position. Each of the three files contains also flags which can be used to mark, e.g., images taken under poor weather conditions or stars used as the
While constructing the database, the grid transformations as well as magnitude zero points were found. The association of a given centroid to the record in the star list was based on the "nearest-to-nearest" algorithm with the maximum fitting radius of one pixel. This approach turned out to be very efficient. During the tests of our software on the data from BLG175 field, we found a star with proper motion as high as 806 mas/yr. Its J2000.0 coordinates are $\alpha = 18^h04^m14^s62$, $\delta = -31^\circ29'28''8$ and parallax is most probably around 50 mas. There were three separate records to which the centroids of this star were assigned and for each of them correct value of the proper motion was found. The databases constructed using the standard OGLE-III reduction pipeline was checked. We remind, that during its construction, the radius used in cross-matching of the star catalog of given image with the reference list was constant. It turned out that none of the standard OGLE-III records of this star gave the correct value of the proper motion, because a significant number of centroids was erroneously assigned. It proves that the cross-matching used here was a robust procedure and allows finding stars with very high proper motions. Such stars can be used to predict future microlensing events (Proft et al., 2011). We note that if a constant radius for cross-matching is used, then there is a limit on the highest proper motions that can be found (e.g., 500 mas/yr in Sumi et al., 2004).

In order to properly set the zero point of the proper motions, one needs a clean sample of bulge stars. We did not want to affect the measured proper motion of RC stars. Thus, to set the proper motion zero point, we used only the stars brighter than $I_0 = 13.8$ mag with color constrains the same as in Fig. 4.1. Additionally, the stars brighter than $I = 16$ mag were used to determine the $r$ vs. $(V - I)$ relation. To all these stars, we fitted the astrometric model without parallax effect (Eqs. 1.3 and 1.4). To obtain the final uncertainties of centroids, we square added the uncertainties of the centroid fitting returned by ASTROWARS and the grid $rms$ found during the database construction. Reciprocal of square centroids uncertainties were used as weights during the grid fitting. The linear fit was performed to $r$ vs. $(V - I)$ relation and the mean proper motion of the reference stars was calculated. From the reference stars, the ones with $\mu > 12$ mas/yr were removed because they were most probably Galactic disk objects. A model with proper motion smaller by the value of the mean proper motion of the reference stars and the differential refraction coefficient was subtracted from the observed centroids. The resulting residuals
were transformed to the coordinates of the raw image and subtracted from the measured positions. Next, the new transformation grids were found. For every reference star the mean position in the reference grid was found. If this position differed by more than 0.15 pix from the reference position, the star was removed from the reference star list. After that, the final grid transformations were found. The models were fitted to all the stars brighter than 16 mag and the final \( r \) vs. \( (V - J) \) relation was found. The last step was fitting models for all the stars in the given database.

We compared the proper motions of stars located in overlapping parts of the adjacent fields. The differences between proper motions found for the same star in two fields turned out to be larger in the right ascension than in the declination. The uncertainties resulting from the covariance matrix were similar in both directions. In order to obtain reliable estimates of the proper motion uncertainties, we performed the bootstrap simulation (Press et al., 1992). In the bootstrap method the input data are randomly drawn with replacement and then are subject to the same analysis as was performed on original data. The procedure is repeated many times and the \( \text{rms} \) of the results gives an estimate of uncertainties. In our case 300 samplings were performed and at the beginning of each one the epochs used were randomly drawn. It clearly showed that the proper motions uncertainties in the right ascension were larger than in the declination. The reason for that was the differential refraction, the parallax, and the proper motion in right ascension all acting in almost the same direction for the bulge fields observed from Las Campanas Observatory (geographic latitude of \( -29^\circ \)). This problem was for the first time pointed out by Eyer & Woźniak (2001). We found significantly larger uncertainties of \( \mu_\alpha* \) in the field BLG160 (see Fig. 4.7). This was caused by the smaller number of observations, which strengthens the degeneracy between \( \mu_\alpha* \), \( r \), and \( \pi \). The proper motions uncertainties were significantly larger close to the subimage borders. This was most probably caused by the smaller number of stars used to fit the grid in these parts of the image.

Our goal was to calculate the proper motion statistics in the galactic coordinates. The proper motions for all stars were transformed to the galactic coordinates. In order to properly transform the proper motion uncertainties, we transformed the proper motions found in every sampling of the bootstrap method and then calculated their \( \text{rms} \). In the galactic coordinates, the longitudinal scale changes with galactic latitude, thus, the proper motion in the galactic longitudinal direction multiplied by the cosine of the galactic latitude
is denoted as $\mu_{i*}$. The proper motion in the galactic latitude is denoted as $\mu_b$. The results for individual stars $i = 1, 2, \ldots$ are designated $\mu_{i*,i}$ and $\mu_{i,b}$ with uncertainties $\sigma_{i*,i}$ and $\sigma_{i,b}$. For the equations defining the conversion from the equatorial system to the galactic one, we refer to Appendix B.

### 4.4 Proper motions of the double red clump

The goal of this section is to estimate the difference in the mean proper motions of both arms of the X-shaped structure and compare the proper motion dispersions. The comparison was performed separately for latitudinal and longitudinal proper motions. We used
the proper motions with uncertainties calculated in Sec. 4.3 and probabilities that given star belongs to one of arms of the X-shaped structure calculated in Sec. 4.1.

To estimate the mean and dispersion of proper motions, we followed the method presented by Rattenbury et al. (2007). The correlations of uncertainties $\sigma_{i,\lambda^*}$ and $\sigma_{i,b}$ were neglected. As an example, we present the equations for proper motion of the brighter arm in the longitudinal direction (subscript $b, \lambda^*$). The corresponding values for the fainter arm as well as the latitudinal direction were calculated using the same method. It is assumed that the proper motions are taken from the normal distribution of the mean $\mu_{b,\lambda^*}$ and dispersion $\sigma_{b,\lambda^*}$. The measured values $\mu_{i,\lambda^*}$ differ from the true values because are subject to measuring errors with zero mean and a dispersion of $\sigma_{i,\lambda^*}$. The likelihood (product of probabilities of obtaining given $\mu_{i,\lambda^*}$) maximizes for values of $\mu_{b,\lambda^*}$ and $\sigma_{b,\lambda^*}$ defined by the following equations:

$$\overline{\mu}_{b,\lambda^*} = \frac{\sum_i \frac{\mu_{i,\lambda^*}}{\sigma^2_{i,\lambda^*} + \sigma^2_{i,\lambda^*}}}{\sum_i \frac{1}{\sigma^2_{i,\lambda^*} + \sigma^2_{i,\lambda^*}}}$$

(4.14)

$$\sum_i \frac{1}{\sigma^2_{b,\lambda^*} + \sigma^2_{i,\lambda^*}} - \sum_i \frac{\left(\mu_{i,\lambda^*} - \mu_{b,\lambda^*}\right)^2}{\sigma^2_{b,\lambda^*} + \sigma^2_{i,\lambda^*}} = 0$$

(4.15)

This equations can be easily solved numerically. For the assumed value of $\sigma_{b,\lambda^*}$ one finds $\overline{\mu}_{b,\lambda^*}$ from Eq. 4.14. This value is used to evaluate the left side of Eq. 4.15. We started with a small and a large values of $\sigma_{b,\lambda^*}$ for which the left side of Eq. 4.15 has negative and positive value, respectively. Then, we used bisection method to find $\sigma_{b,\lambda^*}$ and corresponding $\overline{\mu}_{b,\lambda^*}$. In the case of BLG160 field, the large uncertainties of the proper motions in right ascension prevented evaluation of proper motion dispersions, thus we only calculated mean values of proper motions using $\sigma_{i,\lambda^*}^{-2}$ as weights.

The calculations were performed separately for each subfield, as the reference frames might be slightly different. For each star the probabilities $p_b (I_0)$ and $p_f (I_0)$ were estimated using fits to the LF in given field. The calculations were restricted to the stars lying in the same part of the CMD which were used in the construction of the LF (Fig. 4.1), with proper motion uncertainties $\sigma_{i,\lambda^*} < 2$ mas/yr and $\sigma_{i,b} < 2.2$ mas/yr, as well as $\mu < 10$ mas/yr. Next, we performed the simulation which aim was to find statistical properties of the stellar proper motions in each arm. For each star, a number $q$ between 0 and 1 was drawn from the uniform distribution.\(^5\) The star was assigned to the brighter arm if $q < p_b (I_0)$ or to

\(^5\)Multiplicative congruential random number generator (Press et al., 1992) was used.
Table 4.2: Proper motion differences in fields with a double RC.

<table>
<thead>
<tr>
<th>subfield</th>
<th>$n_{h,\text{eff}}$</th>
<th>$n_{f,\text{eff}}$</th>
<th>$\Delta\mu_r$ [mas/yr]</th>
<th>$\Delta\mu_b$ [mas/yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLG 334.1</td>
<td>456.2</td>
<td>643.1</td>
<td>0.40 ± 0.13</td>
<td>0.01 ± 0.12</td>
</tr>
<tr>
<td>BLG 334.2</td>
<td>464.5</td>
<td>665.3</td>
<td>0.54 ± 0.14</td>
<td>0.02 ± 0.13</td>
</tr>
<tr>
<td>BLG 334.3</td>
<td>482.4</td>
<td>657.1</td>
<td>0.57 ± 0.13</td>
<td>0.04 ± 0.12</td>
</tr>
<tr>
<td>BLG 334.4</td>
<td>484.6</td>
<td>698.0</td>
<td>0.57 ± 0.13</td>
<td>−0.02 ± 0.11</td>
</tr>
<tr>
<td>BLG 334.5</td>
<td>534.1</td>
<td>711.5</td>
<td>0.55 ± 0.14</td>
<td>0.07 ± 0.13</td>
</tr>
<tr>
<td>BLG 334.6</td>
<td>532.2</td>
<td>749.7</td>
<td>0.46 ± 0.12</td>
<td>−0.03 ± 0.12</td>
</tr>
<tr>
<td>BLG 334.7</td>
<td>530.2</td>
<td>723.1</td>
<td>0.49 ± 0.13</td>
<td>−0.04 ± 0.12</td>
</tr>
<tr>
<td>BLG 334.8</td>
<td>487.7</td>
<td>698.3</td>
<td>0.39 ± 0.15</td>
<td>−0.12 ± 0.15</td>
</tr>
<tr>
<td>BLG 334</td>
<td>391.9</td>
<td>5546.0</td>
<td>0.50 ± 0.05</td>
<td>0.00 ± 0.05</td>
</tr>
<tr>
<td>BLG 160.1</td>
<td>496.4</td>
<td>258.1</td>
<td>0.49 ± 0.20</td>
<td>0.11 ± 0.19</td>
</tr>
<tr>
<td>BLG 160.2</td>
<td>517.0</td>
<td>243.0</td>
<td>0.42 ± 0.21</td>
<td>−0.07 ± 0.18</td>
</tr>
<tr>
<td>BLG 160.3</td>
<td>492.5</td>
<td>251.9</td>
<td>0.37 ± 0.21</td>
<td>−0.05 ± 0.18</td>
</tr>
<tr>
<td>BLG 160.4</td>
<td>532.9</td>
<td>257.9</td>
<td>0.39 ± 0.20</td>
<td>0.12 ± 0.28</td>
</tr>
<tr>
<td>BLG 160.5</td>
<td>539.4</td>
<td>269.1</td>
<td>0.55 ± 0.20</td>
<td>−0.14 ± 0.17</td>
</tr>
<tr>
<td>BLG 160.6</td>
<td>534.1</td>
<td>255.4</td>
<td>0.53 ± 0.21</td>
<td>0.04 ± 0.30</td>
</tr>
<tr>
<td>BLG 160.7</td>
<td>557.2</td>
<td>256.6</td>
<td>0.18 ± 0.21</td>
<td>0.26 ± 0.24</td>
</tr>
<tr>
<td>BLG 160.8</td>
<td>495.9</td>
<td>249.5</td>
<td>0.34 ± 0.20</td>
<td>0.06 ± 0.22</td>
</tr>
<tr>
<td>BLG 160</td>
<td>4165.3</td>
<td>2041.5</td>
<td>0.41 ± 0.08</td>
<td>0.01 ± 0.08</td>
</tr>
<tr>
<td>BLG 167.1</td>
<td>833.5</td>
<td>354.4</td>
<td>0.46 ± 0.15</td>
<td>0.11 ± 0.14</td>
</tr>
<tr>
<td>BLG 167.2</td>
<td>852.3</td>
<td>363.2</td>
<td>0.43 ± 0.15</td>
<td>−0.05 ± 0.14</td>
</tr>
<tr>
<td>BLG 167.3</td>
<td>852.3</td>
<td>354.2</td>
<td>0.36 ± 0.15</td>
<td>0.03 ± 0.14</td>
</tr>
<tr>
<td>BLG 167.4</td>
<td>918.9</td>
<td>410.0</td>
<td>0.43 ± 0.14</td>
<td>0.02 ± 0.13</td>
</tr>
<tr>
<td>BLG 167.5</td>
<td>1032.9</td>
<td>448.3</td>
<td>0.48 ± 0.14</td>
<td>−0.01 ± 0.13</td>
</tr>
<tr>
<td>BLG 167.6</td>
<td>965.0</td>
<td>444.6</td>
<td>0.46 ± 0.14</td>
<td>0.07 ± 0.13</td>
</tr>
<tr>
<td>BLG 167.7</td>
<td>924.5</td>
<td>420.3</td>
<td>0.40 ± 0.14</td>
<td>0.03 ± 0.14</td>
</tr>
<tr>
<td>BLG 167.8</td>
<td>883.9</td>
<td>409.0</td>
<td>0.36 ± 0.14</td>
<td>−0.02 ± 0.16</td>
</tr>
<tr>
<td>BLG 167</td>
<td>7263.4</td>
<td>3294.0</td>
<td>0.42 ± 0.05</td>
<td>0.02 ± 0.05</td>
</tr>
<tr>
<td>BLG 176.1</td>
<td>684.8</td>
<td>384.7</td>
<td>0.49 ± 0.15</td>
<td>−0.03 ± 0.14</td>
</tr>
<tr>
<td>BLG 176.2</td>
<td>700.9</td>
<td>374.8</td>
<td>0.59 ± 0.18</td>
<td>−0.05 ± 0.18</td>
</tr>
<tr>
<td>BLG 176.3</td>
<td>698.5</td>
<td>375.4</td>
<td>0.55 ± 0.15</td>
<td>−0.13 ± 0.14</td>
</tr>
<tr>
<td>BLG 176.4</td>
<td>798.5</td>
<td>428.7</td>
<td>0.57 ± 0.15</td>
<td>−0.04 ± 0.14</td>
</tr>
<tr>
<td>BLG 176.5</td>
<td>836.6</td>
<td>464.4</td>
<td>0.43 ± 0.14</td>
<td>−0.04 ± 0.14</td>
</tr>
<tr>
<td>BLG 176.6</td>
<td>847.2</td>
<td>451.3</td>
<td>0.52 ± 0.14</td>
<td>−0.17 ± 0.13</td>
</tr>
<tr>
<td>BLG 176.7</td>
<td>795.4</td>
<td>425.9</td>
<td>0.55 ± 0.14</td>
<td>−0.17 ± 0.14</td>
</tr>
<tr>
<td>BLG 176.8</td>
<td>772.2</td>
<td>424.8</td>
<td>0.36 ± 0.14</td>
<td>−0.05 ± 0.15</td>
</tr>
<tr>
<td>BLG 176</td>
<td>6134.2</td>
<td>3330.1</td>
<td>0.50 ± 0.05</td>
<td>−0.09 ± 0.05</td>
</tr>
</tbody>
</table>

the fainter arm if $q \geq 1 - p_f \left (I_0\right)$. If $p_b \left (I_0\right) \leq q < 1 - p_f \left (I_0\right)$, the star was not assigned to any arm, i.e., it was assumed to be a bulge RG or a foreground MS disk star. Such an assignment scheme ensured that none of the stars was assigned to both arms in a given sampling. After assigning all the stars, we used the Eqs. 4.14 and 4.15 to find the mean and dispersion of the proper motions in both arms and both directions separately. A 1000 samplings of such simulation were performed. Their results were averaged to obtain final results. The uncertainties of the final results were the dispersions of the values obtained in different samplings.

The final results of our simulations are presented in Tab. 4.2 and 4.3. For every subfield
Table 4.3: Proper motion dispersions in fields with a double RC

<table>
<thead>
<tr>
<th>subfield</th>
<th>$\sigma_{b,ls}$ [mas/yr]</th>
<th>$\sigma_{b,bl}$ [mas/yr]</th>
<th>$\sigma_{f,ls}$ [mas/yr]</th>
<th>$\sigma_{f,bl}$ [mas/yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLG134.1</td>
<td>2.59 ± 0.08</td>
<td>2.27 ± 0.08</td>
<td>2.47 ± 0.07</td>
<td>2.16 ± 0.06</td>
</tr>
<tr>
<td>BLG134.2</td>
<td>2.76 ± 0.09</td>
<td>2.44 ± 0.08</td>
<td>2.55 ± 0.07</td>
<td>2.24 ± 0.06</td>
</tr>
<tr>
<td>BLG134.3</td>
<td>2.60 ± 0.08</td>
<td>2.34 ± 0.08</td>
<td>2.43 ± 0.07</td>
<td>2.23 ± 0.06</td>
</tr>
<tr>
<td>BLG134.4</td>
<td>2.62 ± 0.08</td>
<td>2.34 ± 0.08</td>
<td>2.42 ± 0.07</td>
<td>2.08 ± 0.06</td>
</tr>
<tr>
<td>BLG134.5</td>
<td>2.97 ± 0.08</td>
<td>2.63 ± 0.08</td>
<td>2.77 ± 0.07</td>
<td>2.60 ± 0.07</td>
</tr>
<tr>
<td>BLG134.6</td>
<td>2.60 ± 0.08</td>
<td>2.46 ± 0.08</td>
<td>2.46 ± 0.07</td>
<td>2.32 ± 0.06</td>
</tr>
<tr>
<td>BLG134.7</td>
<td>2.67 ± 0.08</td>
<td>2.48 ± 0.07</td>
<td>2.62 ± 0.07</td>
<td>2.26 ± 0.06</td>
</tr>
<tr>
<td>BLG134.8</td>
<td>2.88 ± 0.09</td>
<td>2.81 ± 0.09</td>
<td>2.68 ± 0.07</td>
<td>2.67 ± 0.07</td>
</tr>
<tr>
<td>BLG134</td>
<td>2.70 ± 0.03</td>
<td>2.46 ± 0.03</td>
<td>2.55 ± 0.03</td>
<td>2.29 ± 0.02</td>
</tr>
<tr>
<td>BLG167.1</td>
<td>2.62 ± 0.06</td>
<td>2.43 ± 0.06</td>
<td>2.60 ± 0.10</td>
<td>2.33 ± 0.10</td>
</tr>
<tr>
<td>BLG167.2</td>
<td>2.74 ± 0.06</td>
<td>2.49 ± 0.05</td>
<td>2.72 ± 0.11</td>
<td>2.33 ± 0.09</td>
</tr>
<tr>
<td>BLG167.3</td>
<td>2.64 ± 0.06</td>
<td>2.42 ± 0.05</td>
<td>2.57 ± 0.10</td>
<td>2.30 ± 0.09</td>
</tr>
<tr>
<td>BLG167.4</td>
<td>2.58 ± 0.05</td>
<td>2.46 ± 0.05</td>
<td>2.54 ± 0.09</td>
<td>2.35 ± 0.09</td>
</tr>
<tr>
<td>BLG167.5</td>
<td>2.77 ± 0.05</td>
<td>2.56 ± 0.06</td>
<td>2.73 ± 0.10</td>
<td>2.51 ± 0.09</td>
</tr>
<tr>
<td>BLG167.6</td>
<td>2.71 ± 0.06</td>
<td>2.67 ± 0.05</td>
<td>2.69 ± 0.09</td>
<td>2.56 ± 0.08</td>
</tr>
<tr>
<td>BLG167.7</td>
<td>2.75 ± 0.06</td>
<td>2.63 ± 0.06</td>
<td>2.63 ± 0.10</td>
<td>2.52 ± 0.09</td>
</tr>
<tr>
<td>BLG167.8</td>
<td>2.48 ± 0.06</td>
<td>3.04 ± 0.06</td>
<td>2.47 ± 0.09</td>
<td>2.91 ± 0.10</td>
</tr>
<tr>
<td>BLG167</td>
<td>2.66 ± 0.02</td>
<td>2.57 ± 0.02</td>
<td>2.61 ± 0.04</td>
<td>2.47 ± 0.04</td>
</tr>
<tr>
<td>BLG176.1</td>
<td>2.67 ± 0.06</td>
<td>2.56 ± 0.07</td>
<td>2.59 ± 0.09</td>
<td>2.42 ± 0.09</td>
</tr>
<tr>
<td>BLG176.2</td>
<td>3.00 ± 0.07</td>
<td>3.10 ± 0.08</td>
<td>3.11 ± 0.11</td>
<td>3.04 ± 0.11</td>
</tr>
<tr>
<td>BLG176.3</td>
<td>2.76 ± 0.07</td>
<td>2.50 ± 0.06</td>
<td>2.61 ± 0.10</td>
<td>2.33 ± 0.10</td>
</tr>
<tr>
<td>BLG176.4</td>
<td>2.78 ± 0.06</td>
<td>2.70 ± 0.06</td>
<td>2.74 ± 0.10</td>
<td>2.62 ± 0.09</td>
</tr>
<tr>
<td>BLG176.5</td>
<td>2.85 ± 0.06</td>
<td>2.58 ± 0.07</td>
<td>2.84 ± 0.09</td>
<td>2.48 ± 0.09</td>
</tr>
<tr>
<td>BLG176.6</td>
<td>2.77 ± 0.06</td>
<td>2.62 ± 0.06</td>
<td>2.67 ± 0.09</td>
<td>2.49 ± 0.08</td>
</tr>
<tr>
<td>BLG176.7</td>
<td>2.81 ± 0.06</td>
<td>2.69 ± 0.06</td>
<td>2.64 ± 0.09</td>
<td>2.61 ± 0.09</td>
</tr>
<tr>
<td>BLG176.8</td>
<td>2.67 ± 0.06</td>
<td>2.92 ± 0.07</td>
<td>2.56 ± 0.09</td>
<td>2.74 ± 0.09</td>
</tr>
<tr>
<td>BLG176</td>
<td>2.79 ± 0.03</td>
<td>2.69 ± 0.03</td>
<td>2.70 ± 0.04</td>
<td>2.57 ± 0.04</td>
</tr>
</tbody>
</table>

Results are given for every subfield separately and averaged for each field. The results for the field BLG160 are not shown because of large uncertainties of $\mu_{ls}$.

in Tab. 4.2 presents: the effective number of stars in each arm ($n_{b,\text{eff}} = \sum_i p_b (I_0)$) and $n_{f,\text{eff}} = \sum_i p_f (I_0)$) as well as the difference of the mean proper motions of the brighter and fainter arms ($\Delta \mu_{ls} = \bar{\mu}_{b,ls} - \bar{\mu}_{f,ls}$ and $\Delta \mu_b = \bar{\mu}_{b,b} - \bar{\mu}_{f,b}$). The dispersions of the proper motions in both directions for the two arms separately ($\sigma_{b,ls}$, $\sigma_{b,b}$, $\sigma_{f,ls}$, and $\sigma_{f,b}$) are presented in Tab. 4.3. The results of eight subfields of each field are also averaged and reported in both tables.

We do see that in every subfield (even in BLG134 with $l = -3.2^\circ$) the $\Delta \mu_{ls}$ is close to 0.4 mas/yr and $\Delta \mu_b$ is close to 0. Averaging both quantities across different subfields strengthens the above findings. The proper motion dispersions for the brighter arm are larger than the corresponding values for the fainter arm. This effect is expected if we assume that the linear velocity dispersions in both arms are the same. Similarly to the previous studies of the bulge proper motions, we found the proper motion dispersions in the latitudinal direction larger than in the longitudinal direction (e.g., Rattenbury et al.,
The only existing measurement of $\Delta \mu_l$ and $\Delta \mu_b$ are those performed by McWilliam & Zoccali (2010) using Vieira et al. (2007) proper motions in the Plaut low-extinction window $(l = 0^\circ, b = -8^\circ)$. McWilliam & Zoccali (2010) selected 328 bright RC stars and 365 stars faint RC stars. We suppose that they used all the stars which positions on the CMD matched the brighter and the fainter RC. For the latter this means that only $\approx 1/3$ of the 365 stars actually belonged to the fainter arm of the X-shaped structure. This might have a significant impact on the results. McWilliam & Zoccali (2010) found $\Delta \mu_l = 0.51 \pm 0.18$ mas/yr and $\Delta \mu_b = 0.19 \pm 0.19$ mas/yr. We also found a significant $\Delta \mu_l$ and negligible $\Delta \mu_b$, but our results have significantly smaller uncertainties, were found in different fields ($b \approx 5^\circ$ and a range of $l$), and resulted from the detailed analysis of the stellar associations to the brighter and fainter arm.

There are a few dynamical models of the Galaxy which predict an X-shaped structure (Li & Shen, 2012; Ness et al., 2012; Robin et al., 2012). For neither of the models, proper motion statistics were presented. Dr. Annie Robin kindly provided us with the results of the Besançon model (Robin et al., 2003, 2012) with proper motions for each star given separately. The model input kinematical parameters were not fitted using our results but were taken from the previous works, thus the comparison to the observed values should be rather qualitative than quantitative. The kinematical model was taken from Fux (1999) with model parameters fitted to the results of the BRAVA survey (Rich et al., 2007). The velocity dispersions in $V$ and $W$ directions (positive in the directions of the Galactic rotation and the North Galactic Pole, respectively) are poorly constrained by these data. We tried to analyze the model in the same way as the OGLE-III results. We failed fitting the models to the luminosity functions, as the values of $\sigma_{bRC}/\sigma_{fRC} \approx 2$. It was decided to calculate proper motion statistics using the stars in the two arms based on the distances of the stars. This fact is also an argument to make qualitative comparison instead of quantitative one. The brighter arm was assumed to be between 5.3 kpc and 8.0 kpc, while the fainter one between 8.0 kpc and 10.3 kpc (the limits slightly changed between the fields). In each of the four analyzed fields the 3$\sigma$-clipped mean proper motion and its dispersion were found for both arms. The results are presented in Tab. 4.4. The proper motion differences in longitudinal direction are positive in every field with values ranging from 0.74 mas/yr to 1.01 mas/yr. The latitudinal proper motion differences are much
closer to zero and the most deviating result is $-0.19$ mas/yr. The fact that the values measured by us are closer to zero (Tab. 4.2) is not in conflict with the above findings\(^6\). The comparison of proper motion dispersions (Tab. 4.3) reveals significant discrepancies. We found proper motions dispersions in the brighter arm to be larger than in the fainter arm, while the model gives the opposite predictions. The fainter arm is further from the Galactic disk because of the perspective effect. We note that proper motion dispersions decrease as we move further from the disk between $b = -1.5^\circ$ and $b = -5^\circ$ and increase as we move further than $b = -5^\circ$ from the disk (Rattenbury et al., 2007; Vieira et al., 2007). In our opinion, the discrepancy mentioned above between the predicted and modeled ratios of proper motion dispersions in both arms can be caused by the adopted model parameters which result in different $b$ at which proper motion dispersions start to increase. In each field the longitudinal dispersion is larger than the latitudinal one. This agrees with our findings as well as the previous measurements and model predictions.

Table 4.4: Proper motion statistics predicted by the Besançon model

<table>
<thead>
<tr>
<th>field</th>
<th>$\Delta \mu_*$ [mas/yr]</th>
<th>$\Delta \mu_b$ [mas/yr]</th>
<th>$\sigma_{b,\ell}$ [mas/yr]</th>
<th>$\sigma_b$ [mas/yr]</th>
<th>$\sigma_{f,\ell}$ [mas/yr]</th>
<th>$\sigma_{f,b}$ [mas/yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLG134</td>
<td>1.01</td>
<td>-0.19</td>
<td>2.75</td>
<td>2.24</td>
<td>3.18</td>
<td>2.64</td>
</tr>
<tr>
<td>BLG160</td>
<td>0.99</td>
<td>-0.08</td>
<td>2.60</td>
<td>2.38</td>
<td>2.91</td>
<td>2.61</td>
</tr>
<tr>
<td>BLG167</td>
<td>0.75</td>
<td>-0.10</td>
<td>2.65</td>
<td>2.42</td>
<td>3.30</td>
<td>2.83</td>
</tr>
<tr>
<td>BLG176</td>
<td>0.74</td>
<td>-0.07</td>
<td>2.65</td>
<td>2.41</td>
<td>3.05</td>
<td>2.60</td>
</tr>
</tbody>
</table>

We plan to publish a full catalog of the stellar proper motions in the OGLE-III bulge fields with the proper motion dispersions found in all fields. As a check, we performed such an analysis of the field BLG173 which has a much better observing coverage for the proper motions calculations than the fields with a double RC. The results are presented in Tab. 4.5. The uncertainties of the RC dispersions $\sigma_{\ell}$ and $\sigma_b$ are as small as $0.03$ mas/yr and gave consistent results in the neighboring subfields. This shows that our measurements in other fields would be very useful for constraining the models of the bulge kinematics with better accuracy, spatial resolution, and sky coverage than in Rattenbury et al. (2007). In order to compare our results with the previous studies, we have searched for the proper motion dispersions in the bulge (Spahnhauer et al., 1992; Zoccali et al., 2001; Feltzing & Johnson, 2002; Kuijken & Rich, 2002; Kozłowski et al., 2006; Rattenbury et al., 2007;\(^6\) The proper motions of stars which had significant $p_0(I_b)$ and $p_f(I_b)$ reduced the measured values of $\Delta \mu_*$ and $\Delta \mu_b$. }
Vieira et al., 2007), which were found in fields close to ours. These fields are given in Tab. 4.6. It turned out that only field BLG176 is close to fields analyzed in the literature. Our results in this field are in general agreement with the closest fields of Kozłowski et al. (2006) and Rattenbury et al. (2007).

Table 4.5: Proper motion dispersions in BLG173

<table>
<thead>
<tr>
<th>subfield</th>
<th>$n_{\text{eff}}$</th>
<th>$\sigma_{l}$ [mas/yr]</th>
<th>$\sigma_{b}$ [mas/yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLG173.1</td>
<td>2631.8</td>
<td>2.92 ± 0.03</td>
<td>2.59 ± 0.03</td>
</tr>
<tr>
<td>BLG173.2</td>
<td>2611.5</td>
<td>2.88 ± 0.03</td>
<td>2.73 ± 0.03</td>
</tr>
<tr>
<td>BLG173.3</td>
<td>2605.0</td>
<td>3.03 ± 0.03</td>
<td>3.62 ± 0.04</td>
</tr>
<tr>
<td>BLG173.4</td>
<td>2818.6</td>
<td>2.97 ± 0.03</td>
<td>2.75 ± 0.03</td>
</tr>
<tr>
<td>BLG173.5</td>
<td>3144.2</td>
<td>3.09 ± 0.03</td>
<td>2.91 ± 0.03</td>
</tr>
<tr>
<td>BLG173.6</td>
<td>2949.1</td>
<td>3.11 ± 0.03</td>
<td>3.85 ± 0.04</td>
</tr>
<tr>
<td>BLG173.7</td>
<td>2905.6</td>
<td>2.88 ± 0.03</td>
<td>2.83 ± 0.03</td>
</tr>
<tr>
<td>BLG173.8</td>
<td>2896.8</td>
<td>2.78 ± 0.03</td>
<td>2.67 ± 0.03</td>
</tr>
</tbody>
</table>

Table 4.6: Bulge proper motion dispersions by other authors.

<table>
<thead>
<tr>
<th>reference</th>
<th>field</th>
<th>$l$ [°]</th>
<th>$b$ [°]</th>
<th>$\sigma_{l}$ [mas/yr]</th>
<th>$\sigma_{b}$ [mas/yr]</th>
<th>closest field from Tab. 2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>K06</td>
<td>120-A</td>
<td>1.76</td>
<td>−4.48</td>
<td>2.75 ± 0.09</td>
<td>2.52 ± 0.09</td>
<td>BLG176</td>
</tr>
<tr>
<td>R07</td>
<td>6</td>
<td>−0.25</td>
<td>−5.70</td>
<td>2.61 ± 0.02</td>
<td>2.36 ± 0.03</td>
<td>BLG160, BLG176</td>
</tr>
<tr>
<td>R07</td>
<td>46</td>
<td>1.09</td>
<td>−4.14</td>
<td>2.90 ± 0.04</td>
<td>2.67 ± 0.04</td>
<td>BLG176</td>
</tr>
</tbody>
</table>

K06–Kozłowski et al. (2006), R07–Rattenbury et al. (2007).
Chapter 5

Conclusions

The OGLE-III has collected a very rich observing material. Its main goal was high precision photometry of a large number of stars in a dense stellar fields of the Galactic bulge and Magellanic Clouds. Both the observing strategy and the data reduction pipeline were optimized in order to give high accuracy photometry. We used the same images to perform precise astrometric measurements. First, we analyzed the Magellanic Clouds fields. We corrected the stellar centroids measured using the DoPHOT software. Based on the corrected centroids, we prepared a catalog of over 6.2 million stellar proper motions. The separate analysis was performed for selected fields in the Galactic bulge. The data reduction was conducted using our software written in order to precisely measure centroids. Below, we summarize the obtained results.

In the Magellanic Clouds fields, the clean and highly complete list of 549 HPM stars (i.e., with $\mu > 100$ mas/yr) was prepared. It contains the closest stars in this part of the sky. There are several dozens of WDs and one definite subdwarf on this list.

The catalog of all stellar proper motions that could be reliably derived in the Magellanic Clouds was also presented. Parallax estimates were given for over 110 000 stars. The usage of the catalog in different astrophysical contexts were presented. A few hundreds of the WDs were selected using the $(V - I)$ color and both the RPM as well as the absolute brightness calculated using parallaxes. The CPM binaries were selected using statistical properties of the sample. They were further analyzed using the RPM diagram. The most interesting CPM binaries found include a double WD system. The coincidence of the SMC and the globular cluster 47 Tuc on the sky gave us an opportunity to conduct research not possible otherwise. The mean proper motion of the cluster against the SMC was measured...
with an uncertainty of 0.6 mas/yr per coordinate. Unexpected usage of the catalog was a
search for stars that escaped from the gravitational potential of the cluster and now form
tidal tails. A list of promising candidates for such stars was prepared and it is pending
spectroscopic verification. Three new cluster variable stars were identified. We also tried
to measure the absolute proper motion of the Magellanic Clouds. It is limited by the
number of bright and isolated quasars known behind the LMC and SMC. The proper
motions of variable stars were separately discussed. We have shown that suspected group
of curious blue LMC variables in fact contains the Galactic foreground stars. Finally, we
have presented how the proper motion catalog can be used to map the classical Cepheid
instability strip.

The other environment in which the proper motions were measured and analyzed was
the Galactic bulge. Four fields were selected which fulfill two criteria: show a double
RC in the CMD and have large enough number of epochs to allow measuring proper
motions precisely. Using our own software, the reduction of all images collected by the
OGLE-III in these fields was performed. The results were used to measure the proper
motions. The tests revealed that our algorithm is capable to effectively discover stars with
proper motions as high as 800 mas/yr. The probabilities that a given star belonged to
the brighter or fainter arm of the X-shaped structure were calculated using the LF of the
dereddened magnitudes. Using these probabilities and proper motions, we compared the
proper motions of the stars in two arms of the X-shaped structure. Significant differences
in longitudinal proper motions of \( \approx 0.5 \) mas/yr were found. The differences in latitudinal
proper motions turned out to be consistent with zero. For the first time, we measured the
proper motion dispersions separately for both arms of the X-shaped structure. We found
that the proper motion dispersions in the brighter arm were larger than in the fainter one.
These results can be used to constrain dynamical models of the Galaxy. The preliminary
comparison with predictions of one such a model was performed.

The accuracy of measured proper motions is below 0.5 mas/yr for bright stars. The
parallaxes were measured with uncertainties below 2 mas. This shows that the long-
term ground-based observing project conducted on a 1–2 m class telescope, such as the
OGLE-III, can be used to find astrometric parameters of large number of stars with high
accuracy. It was shown that a large catalog of stellar proper motions can be used in a
variety of research areas.
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Appendix A

Software description

The AstroWars stores the PSF as a tabulated function which is defined on a grid of points. The exact size of the PSF grid depends on the image seeing, what gives this software the flexibility needed to analyze the OGLE-III data. After reading the input FITS-type image, the program masks all the bad pixels i.e., the ones with values below 50 ADU or above 60000 ADU (i.e., overexposed pixels) and the bad pixels supplied by the user. The bad columns of the OGLE-III camera were tabulated and are also masked. The masked pixels are not used in the further reduction. Next, the average sky background (sky) and its dispersion ($\sigma_{sky}$) are found. First, the eight pixels neighboring the ones with values below 50 ADU or above 60000 ADU are marked. Then, the median value ($med_1$) of the rest of the image pixels is found. The pixels with values below $med_1$ are taken to calculate the root mean square ($rms_1$) of the differences between them and $med_1$. Next, the pixels with values larger than $med_1 - 2 \cdot rms_1$ and smaller than $med_1 + 2 \cdot rms_1$ are taken into account. Their median ($med$) and average ($ave$) are found. Using these quantities the sky background is derived. If $med < ave$, then we use the same formula as Stetson (1987):

$$sky = 3 \cdot med - 2 \cdot ave$$

(A.1)

otherwise, we assume sky background to be equal to $med$. The sky dispersion $\sigma_{sky}$ is calculated using only pixels with values between $sky - 3 \cdot rms_1$ and $med_1 + 2 \cdot rms_1$. From the pixel values in this interval the $sky$ value is subtracted. The $rms$ of the results is taken as an estimate of $\sigma_{sky}$. Our tests revealed that this method of $sky$ and $\sigma_{sky}$ estimation gives reliable results for a wide range of the $sky$ values and the surface densities of stellar
images. The value of sky is subtracted from every image pixel to make further analysis easier.

After finding the average sky value and its dispersion, AstroWars searches for the star-like objects. It is well known that the optimal method to search for star-like objects is to convolve the image with a Gaussian profile which dispersion is close to the one of the image PSF (Irwin, 1985; Stetson, 1987). If the assumed dispersion of the Gaussian kernel does not differ by more than 30%, the detection efficiency is not significantly worse. We note that the convolution is a time-consuming calculation. In order to have both good detection rate and program running on reasonable timescales, we decided to fix the convolution kernel to a Gaussian function with a dispersion corresponding to FWHM = 5.5 pix. The local maxima of the convolved image are selected. For each of them the value of the corresponding fitted Gaussian function is estimated. As the star-like objects we consider the maxima which fulfill three requirements. First, the value of the fitted Gaussian has to be above the selected limit and below 60,000 ADU. Second, the value of the fitted Gaussian divided by the image value at the same pixel is between 1/3 and 3. Third, none of the eight direct neighbors of the local maximum was marked as a bad pixel. These criteria intended to exclude cosmic rays, pixels close to bad columns of the CCD frame etc.,. To find the first estimate of the stellar centroid, a parabola is fitted to the maximum of the convolved image and two nearest points (for each of the image directions separately). The smallest and largest shifts from the maximum of the convolved image are fixed to −0.5 or 0.5.

An important parameter of the image, which is needed in further analysis, is the FWHM of the PSF. For every star we found the FWHM in two directions. The median of the results was taken as the image FWHM. In further analysis, if the dispersion of the Gaussian fit to the PSF is needed, it is taken from the FWHM found i.e., \( \sigma = \text{FWHM}/2.35 \).

The next very important step in the image reduction is choosing the PSF stars i.e., the stars which would be used to find the PSF. The stars at least five times brighter than the detection threshold are selected. The ones located very close to the image edges were not considered. The list of the PSF stars includes objects located in the wings of the very bright stars. Such stars cannot be used for the PSF fitting. To remove such stars from the PSF stars, the average value of pixel counts is calculated in concentric rings around the stellar centroid. If the ring which is further from the centroid has higher average value
than the ring closer to the centroid, the star is removed from the PSF star list. The next step of the PSF stars verification, is similar to the one used in DoPHOT. For each star the marginal sums are calculated and Gaussian functions are fitted. If the extremum value of the fitted Gaussian is negative or much above the maximum possible number counts, the star is removed from the PSF star list. When the list of PSF the stars is constructed it is split into parts corresponding to the image subframes (see Fig. 4.4). Up to 60 brightest stars were assigned to each subframe. If the images of two bright stars overlap, then the fainter one is not taken into account.

After finding the list of the PSF stars in every region, we can find the first approximation of the PSF. For every pixel in the closest neighborhood of the star, we calculate its position relative to the stellar centroid and normalize the pixel value by the stellar flux. Every such pixel gives one sampling of the PSF. The PSF value is found separately in each grid point. To calculate it, all the samplings which coordinates do not differ by more than 0.25 pixel are taken into account (see Fig. 5 in Anderson & King, 2000)\textsuperscript{1}. The PSF value in the given grid point is a 2\sigma-clipped average of the selected samplings.

Each time, the new approximation of the PSF is found a few additional calculations to fine-tune the results are performed. First, the PSF is smoothed by replacing each value with the average of the $5 \times 5$ pixel square. Such a crude smoothing method is used only in the first step. Second, the PSF is normalized in order to compensate small differences of the sum of the PSF grid points values and unity. Third, one has to account for unavoidable errors in centroids of the PSF stars. To correct for them, the PSF is fitted with a 2D-Gaussian function which has five free parameters: mean $x$ and $y$ values, two dispersions and the correlation coefficient. The fit is performed using the downhill simplex method (Press et al., 1992). The mean $x$ and $y$ coordinates of the fitted Gaussians are divided by four (i.e., by the super-sampling factor) and subtracted from the positions of all the stars. The idea that the PSF centroiding can be achieved by shifting the coordinates of the PSF stars not by shifting of the PSF itself was presented by Anderson & King (2000).

The next step is the second adjusting of the PSF model. To find the first approximation of the PSF, we use the value of every sampling. While finding the second approximation and all the following ones, instead of the value of every sampling, we use the difference between the value and the PSF value evaluated using the previous PSF approximation.

\textsuperscript{1}In this approach every sampling contributes to four grid points which are closest to it.
These differences are subject to the same procedure as described above i.e., for every grid point the samplings which coordinates do not differ by more than 0.25 pixel are subject to calculation of a $2\sigma$-clipped average. The average values for all grid points represent the correction of the PSF. This correction is added to the previous PSF approximation to obtain the next one.

During the above calculations, we have to evaluate the PSF at a given position of the frame and a given offset from the centroid for the first time. Thus, we describe the details of this procedure. To find the PSF at a given location of the frame the four nearest subregion PSF are used (see Fig. 4.4). Each grid point is evaluated by bi-linear interpolation of the grid points with the same offset from the centroid of the four subregion PSFs. The result of the interpolation is a super-sampled PSF in the given frame location. To evaluate the value of the PSF at a given offset from the centroid, we use the bi-cubic spline interpolation of the four nearest grid points. To calculate it, one needs function value, its first derivatives in the $x$ and $y$ directions, as well as the $xy$ cross derivative. The derivatives are calculated as follows:

$$\frac{\partial \text{PSF}_{i,j}}{\partial x} = \frac{\text{PSF}_{i+1,j} - \text{PSF}_{i-1,j}}{2}$$  \hspace{1cm} (A.2)

$$\frac{\partial \text{PSF}_{i,j}}{\partial y} = \frac{\text{PSF}_{i,j+1} - \text{PSF}_{i,j-1}}{2}$$  \hspace{1cm} (A.3)

$$\frac{\partial^2 \text{PSF}_{i,j}}{\partial x \partial y} = \frac{\text{PSF}_{i+1,j+1} - \text{PSF}_{i+1,j-1} - \text{PSF}_{i-1,j+1} + \text{PSF}_{i-1,j-1}}{4}$$  \hspace{1cm} (A.4)

Where $\text{PSF}_{i,j}$ is the PSF value at the point offset by $(i, j)$ from the centroid. This way of finding PSF for a given star is also used in all further calculations.

Similarly to the first iteration, we smooth and normalize the model PSF. We use the $5 \times 5$ quadratic Savitzky-Golay smoothing filter\textsuperscript{2}. This filtering is used for the grids points closest to the centroid. The remaining pixels are smoothed in the same way as in the first iteration. In all successive iterations we use the same smoothing algorithm.

In order to center the PSF model, the five-parameter Gaussian function is again fitted to the PSF in every subframe. The resulting coordinates of the Gaussian center are linearly interpolated between the subregions and the results are subtracted from the PSF stars centroids.

\textsuperscript{2}This filter is equivalent to identity for quadratic functions. Anderson & King (2000) presented its kernel in their Eq. 8 with typing errors. Instead of $-0.078368$ there should be $0.078368$ and $-0.081816$ should be replaced by $-0.080816$. 
The PSF model is once more updated, smoothed and normalized. The result is used to fit all stars in the image with the new PSF. The fit has three free parameters—the $x$ coordinate, the $y$ coordinate and the stellar flux. The flux can be estimated using linear regression of the PSF and the pixel values. This allows using the downhill simplex method in two dimensions to minimize the $\chi^2$ of pixel values in pixels corresponding to a given star. We note that every evaluation of the $\chi^2$ requires the PSF model for a star centered at slightly different coordinates. Thus, we have to perform bi-cubic spline interpolation of the PSF at least a few dozens times for one star. The bi-linear interpolation of the four nearest PSF subregions is performed only once for a given star.

At this stage we have the PSF model as well as the stellar fluxes and centroids good enough to erase the stars from the image. The method of removing the stars in order to constrain PSF better is well known and was already used by Stetson (1987). To do this, we evaluate the PSF for every star, multiply it by the stellar flux and subtract the resulting pixel values from the corresponding image pixels. After removing all the stars we are left with the erased image. It is used to evaluate sky background for every star separately. The pixels in a ring around the star are selected and their median value is taken as a sky background estimate. The individual value of the sky background for every star is particularly needed when analyzing images with high stellar density, because it is not possible to fully subtract all the faint sources that appeared in the image. The OGLE-III bulge images analyzed here certainly needed separate sky value for every star.

After calculating the sky values for all stars, we again correct the stellar positions. Two important differences have to be stressed at this point. First, the sky value of every individual star is taken into account during fitting. Second, the fit is performed in the image with all the stars (except the fitted one) erased. After correcting all the stars in such a way, we remove all the stars except the PSF ones using the centroids and fluxes found a while ago. Such an image is used to adjust the PSF model, which is next smoothed and normalized. The PSF centers are calculated and subtracted from the PSF stars positions. Next, we adjusted the PSF model, smoothed it and normalized.

At this stage we have already corrected the centroid of each star a few times. If two blended stars are fitted, it might have happened that the measured centroids of the two stars became very close to each other, even closer than the smallest resolvable separation allowed by the image seeing. From each pair of stars, which are closer to each other than
half of the PSF dispersion, we remove the star from the list of stars and updated the record for the second one, so that it best fits to that part of the image.

At this point one can search for faint stars using the convolution of the erased image with the PSF model found. We have not implemented this fully. We once more update the sky value for every star and correct the stellar centroids and fluxes. Next, we erase all the stars except the PSF once. This is followed by the PSF model adjustment, its smoothing and normalizing. The PSF centers are found for the last time and PSF stars centroids are corrected accordingly. These are used for the final PSF model adjustment. It is smoothed and normalized. The final fit of stellar centroids is performed using the derived PSF.

The brightness uncertainties are found using the residual pixel counts for every star. In order to save disk space, we decided to give common uncertainty of the centroid fitting common for both coordinates. We use the modified version of equation given by Kuijken & Rich (2002):

$$
\sigma_{\text{cent}} = \frac{\sigma_f}{f} \sqrt{\frac{1}{2} \left( \frac{1}{\int (\partial PSF/\partial x)^2 dxdy} + \frac{1}{\int (\partial PSF/\partial y)^2 dxdy} \right)}
$$  \hspace{1cm} (A.5)

where \( f \) and \( \sigma_f \) are flux and its uncertainty, while \( \sigma_{\text{cent}} \) is the uncertainty of the centroid.

The Astrowars software outputs the main diagnostic statistics of calculations, the final list of stars, the PSF model and the erased image if required. The list of stars can be either given in text file or binary file to save disk space.

We plan to make Astrowars publicly available in a near future. We hope that the growing number of CCD images taken with a wide field cameras would be used for precise astrometry and our software would be used for that purpose.
Appendix B

Conversion of the sky coordinates and proper motions

The conversion of the coordinates between equatorial system \((\alpha, \delta)\) and the galactic one \((l, b)\) is described by the following equations:

\[
b = \arcsin (\cos \delta \cos \delta_G \cos (\alpha - \alpha_G) + \sin \delta \sin \delta_G)
\]

\[
l = \arctan \left( \frac{\sin \delta - \sin b \sin \delta_G}{\cos \delta \sin (\alpha - \alpha_G) \cos \delta_G} \right) + l_\Omega
\]

The \(\alpha_G\) and \(\delta_G\) are the equatorial coordinates of the north galactic pole. The \(l_\Omega\) denotes the galactic longitude of the ascending node of the galactic plane. The values of these quantities are following Perryman & ESA (1997):

\[
\alpha_G = 192^\circ 85948
\]

\[
\delta_G = 27^\circ 12825
\]

\[
l_\Omega = 32^\circ 93192
\]

By differentiating the Eq. B.1 and B.2 and substituting \(\mu_b = db/dt\), etc. we obtain:

\[
\mu_b = \frac{\mu_{\delta} C_1 - \mu_{\alpha\star} C_2}{\cos b}
\]

\[
\mu_l = \frac{\mu_{\delta} C_3 - \mu_{\delta \star} C_4 - \mu_{\alpha\star} C_5}{\cos \delta \sin (\alpha - \alpha_G) \cos \delta_G (1 + \tan^2 (l - l_\Omega))}
\]
Where the coefficients $C_1$, $C_2$, $C_3$, $C_4$, and $C_5$ are defined as follows:

\[
C_1 = \sin \delta_G \cos \delta - \cos \delta_G \sin \delta \cos (\alpha - \alpha_G) \\
C_2 = \cos \delta_G \sin (\alpha - \alpha_G) \\
C_3 = \cos \delta + \tan (l - l_\Omega) \cos \delta_G \sin (\alpha - \alpha_G) \sin \delta \\
C_4 = \cos b \sin \delta_G \\
C_5 = \cos \delta_G \tan (l - l_\Omega) \cos (\alpha - \alpha_G)
\]
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