Undoubtedly the most popular family of means used in mathematical analysis, statistics, probability and other branches of mathematics are Power Means. In the late 1920's and in the beginning on 1930's Kolmogorov, Nagumo and de Finetti, independently, proposed the new family being the generalization of this family – currently adopted for the name of quasi-arithmetic means. These means are defined by the equality $f^{-1}(\sum f(a_i)/n)$, where f is a continuous, strictly monotone function defined on the interval, while $(a_i)_{i=1}^n$ is a vector of arguments. For such objects, there naturally appear a whole list of questions regarding the adaptation of the classical results known for Power Means.

An example of such a problem is to adopt the classical fact, well-known for Power Means, claiming that for any fixed vector of arguments, as the parameter change among all possible arguments, one obtains (exactly once) all the intermediate values between the smallest and the largest component of the vector. In my thesis I make an attempt to resolve, using - it seems - quite advanced methods, a question when a family of quasi-arithmetic means has this property (so-called scale property).

Another important issue is the question how does a small change of the function f affecting the value of quasi-arithmetic mean generated by f. Some results in this area were obtained already in the 1960s by Cargo and Shisha (however, some additional conditions concerning regularity were done). The problem of finding necessary and sufficient conditions for convergence in the family of quasi-arithmetic means (not giving any estimate of the distance) was solved by Pàles in the late 1980's. My results provide new estimates referring to the result of Cargo and Shisha and, at the same time, generalizing Pàles'es result.

Another class of problems studied in my dissertation is a list of questions related to the Hardy means. Its history was started by Hardy's result from 1920 - the answer to a previous Hilbert's question from 1909. Hardy proved that if $\mathcal{P}_p$ is a p-th order power mean, $p \in (0,1)$ and $(a_i)_{i=1}^\infty \in l^1(\mathbb{R}_+)$ then $\sum_{n=1}^\infty \mathcal{P}_p(a_1,\dots,a_n) < (p-p^2)^{-1/p} \sum_{n=1}^\infty a_n$. (One year later, Landau obtained a result with the optimal constant at the right hand side.) This was, however, only a starting point for further research - currently a mean M is called *Hardy* if there exists a constant $C>0$ such that

$$\sum_{n=1}^\infty M(a_1,\dots,a_n) < C \sum_{n=1}^\infty a_n \text{ for any sequence } a \in l^1(\mathbb{R}_+).$$

A natural question is whether some particular mean is Hardy. In the present thesis, I an going to prove this property for several families of means, as well as give a lot of negative results regarding having

Hardy property. Among the already obtained results are the necessary and sufficient condition for a family which is a generalization of the arithmetic-geometric mean, considered by Gauss, and solution of hypotheses established in 2004 by Páles and Persson.