The problem of how to fairly divide a surplus obtained through cooperation is one of the most fundamental issues studied in coalitional game theory. It is relevant to a wide range of economic and social situations, from sharing the cost of a local wastewater treatment plant, through dividing the annual profit of a joint venture enterprise, to determining power in voting bodies. Assuming that the coalition of all the players (i.e. the grand coalition) forms, Shapley [20] defined a unique division scheme, called the Shapley value, which satisfies four intuitive axioms: it distributes all payoff among the players (Efficiency) in a linear way (Additivity) while treating symmetric players equally (Symmetry) and ignoring players with no influence on payoffs (Null-Player Axiom).

The Shapley value has been originally defined for classical coalitional games, which are built on the following two simple assumptions: every group of players is allowed to form a coalition and performance of every coalition is to be rewarded with a real-valued payoff being completely independent from the performance or payoffs of other coalitions. However, while in certain cases the simplicity is the strength of the classical coalitional game model, it often becomes a limitation. Indeed, the classical model is too simple to adequately represent various real-life situations.

The goal of this thesis is twofold:

- **Part I. Extension of the Shapley value to games with externalities**

A number of extensions of the Shapley value have been proposed in the literature. One especially vivid direction of research is how to extend the classical coalitional game model so to account for externalities from coalition formation. Such externalities, as formalized by Thrall and Lucas [25], occur in all the situations where the value of a group depends not only on its members, but also on the arrangement of other players. In other words, there is an external impact on the value of a group. As a matter of fact, externalities are common in many real-life situations, e.g., the merger of two companies affects the profit of its competitor. Unfortunately, in such settings four axioms proposed by Shapley are not enough to imply a unique value. For the last fifty years the problem how to extend Shapley value to games with externalities has not been resolved. This issue is the focus of Part I of our work.
• **Part II. Algorithms for the Shapley value for graph-restricted games**

In Part II we depart from games with externalities and study games on graphs. In the model proposed by Myerson [16] called *graph-restricted games* agents (or players) can communicate and cooperate only with agents that they know or are connected to. Such restrictions emerge in social networks analysis, but also in sensor networks, telecommunications or trade agreements, which makes it one of the most recent application of coalitional games. Now, if we extend the game defined only for connected groups to full coalitional game, then by calculating Shapley value we can obtain a value of a player in graph-restricted game. Unfortunately, calculating Shapley value in general requires enumerating of all $2^n$ coalitions. Designing an efficient algorithm for the graph-restricted games is the goal of the second part of the thesis.

**Coalitional games**

The classic model of coalitional games is as follows. Let $N$ be the set of players. By a *coalition* (denoted by $S$) we mean any non-empty subset of $N$. Now, a *game* is given by a function $v$ that associates a real value with every coalition, i.e., $v : 2^N \to \mathbb{R}$. As is customary in the literature, we assume that the coalition of all players (i.e., grand coalition) will form. Then the outcome of the game (or the *value of the game*) is some distribution of jointly achieved payoff $v(N)$ between them – $\phi$ denotes a vector of payoffs and $\phi_i$ is the share of player $i$. Now, the most important normative division scheme was proposed by Shapley:

$$SV_i(v) = \sum_{S \subseteq N, i \in S} \frac{(|S| - 1)!(|N| - |S|)!}{|N|!} \left( v(S) - v(S \setminus \{i\}) \right).$$

(1)

To introduce games with externalities we need few additional definitions. A *partition* of players (denoted $P$) can be formalized as a partition of $N$, that is, a set of disjoint coalitions whose union is $N$. Now, a pair $(S, P)$, where $P$ is a partition of $N$ and $S \in P$, is called an *embedded coalition*. The set of all partitions and the set of all embedded coalitions over $N$ are denoted by $\mathcal{P}(N)$ and $\mathcal{EC}(N)$.

In the basic definition of a coalitional game a value is assigned to every coalition. That way it can only model environments in which coalition have the same value no matter how other players are arranged, i.e., what coalitions they form. To model externalities we introduce *games in partition-function form*: here function $v$ associates a real number with every embedded coalition, i.e., $v : \mathcal{EC}(N) \to \mathbb{R}$.

**Part I. Extension of the Shapley value to games with externalities**

A natural requirement for a fair division scheme is that it remunerates the players of a coalitional game based on their *contribution* to the surplus generated through cooperation. For example, in Shapley’s axiomatization, the Null-Player Axiom requires that no payoff will be allocated to players that make zero contribution to any possible coalition in the game:

**Null-Player Axiom:** if $\forall S \subseteq N, j \in smc_j(S) = 0$ then $\phi_j(v) = 0$ for every game $v$ and agent $i \in N$. 


where $mc_i(S)$ denotes the contribution of player $i$ to coalition $S$. The key issue, then, is how such a contribution should be measured.

In the context of cooperative games, the marginal contribution of a player to a coalition is the difference between the value of this coalition with and without the player:

$$mc_i(S) \overset{\text{def}}{=} v(S) - v(S \setminus \{i\}).$$

It can be also understood as a loss incurred by the remaining players should the player leave a given coalition. Considering this latter intuition, the Shapley value is defined as the average marginal contribution of a player, taken over all possible ways to dissolve the grand coalition by removing players one after the other in a queue (i.e. permutation) until the empty coalition is obtained. In any given permutation, the marginal contribution of a particular player is assigned deterministically as it does not play a role in what a player does after leaving a coalition. This is, however, not the case in games with externalities, where the definition of the marginal contribution becomes much more intricate.

When externalities are present, the value of the coalition that a player has left may be influenced by which coalition, if any, this player subsequently joins. In other words, the choice of a player’s action after it leaves a coalition may result in different values of the player’s marginal contribution to that coalition. One way to account for all such values is to assume that a player can choose to join different coalitions with different probabilities—we will denote the set of such probabilities (or weights) by $\alpha$:

$$[mc^\alpha_i(v)(S,P) \overset{\text{def}}{=} \sum_{T \in P \setminus \{S\}} \alpha_i(S \setminus \{i\}, \tau_T^i(P))[v(S,P) - v(S \setminus \{i\}, \tau_T^i(P))],$$

where $\tau_T^i(P)$ denotes the partition obtained by the transfer of agent $i$ to coalition $T$ in partition $P$. Then, in games with externalities, the sequential dissolution of the grand coalition according to a given permutation of players can be viewed as a stochastic process, rather than a deterministic one. The marginal contribution of a player is then the difference between the value of the coalition with the player and the expected value of this coalition when the player has left.

In games with externalities, not only the definition of the marginal contribution but also the axiomatization of the value becomes more involved, and it can be easily shown that the standard translation of Shapley’s axioms to games with externalities does not yield a unique value. A number of methods have been developed in the literature to address this issue. Some, such as [4] and [10], obtain uniqueness by modifying some of Shapley’s original axioms. Other contributors add new axioms (and sometimes drop some of the original ones), moving increasingly further away from Shapley’s original axiomatization. For instance, Grabisch and Funaki used Markovian and Ergodic Axioms and modified the Symmetry and the Null-Player Axioms [7]. Yet another method is to build extensions to games with externalities relying on alternative axiomatizations of the original Shapley value, such as Myerson’s [17] balanced-contribution axiomatization or Young’s [26] monotonicity axiomatization.

**Results:** In our work, we focus on the first method mentioned above—we study how Shapley’s original axiomatization can be adapted to games with externalities using marginal contributions parametrized with $\alpha$-weights:

**$\alpha$-Null-Player Axiom:** if $\forall (S,P) \in EC(N), \forall i \in S [mc^\alpha_i(v)(S,P) = 0$ then $\phi_i(v) = 0$ for every game in a partition-function form $v$ and agent $i \in N$. 

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We will refer to this approach as the *marginality approach*.

To date, a few definitions of $\alpha$-weights have been proposed in the literature [4, 10, 3]. Here, authors proved that the corresponding strengthening of Shapley’s axiomatization yields a unique value. As a case study, in this thesis we introduce a new definition of $\alpha$-weights, called the *Steady Marginality*, and show an analogous uniqueness.\(^1\)

W sposób naturalny pojawia się zatem pytanie czy osobna osiągana jest unikalność dla innych wag oraz czy podwójna marginalnego mogą użyć także do innych wartości.

However, these results apply only for a specific $\alpha$-weights. This naturally raises two questions: whether the uniqueness can be obtained for other weights and to what extent marginality approach can be used to capture other values. Until now, the most general result in this spirit was obtained for the third method: Fujinaka [5] proved that, for any $\alpha$, Young’s monotonicity axiomatization, parametrized with $\alpha$, guarantees a unique value. However, no such study for Shapley’s original axiomatization exists in the literature.

We begin by proving that, for every value of $\alpha$, Shapley’s original axioms parametrised by $\alpha$ yield a unique extension of the Shapley value for games with externalities.\(^2\)

**Theorem 1.** There exists a unique value that satisfies Efficiency, Symmetry, Additivity and the $\alpha$-Null-Player Axiom for every $\alpha$.

We will refer to this value as the $\alpha$-value. The results of [4, 10], focusing on two particular sets of weights $\alpha$, can be considered as special cases of this general theorem. Furthermore, the theorem is a counterpart of Fujinaka’s result for Young’s axiomatization [5]. We then extend the analysis of the marginality approach as follows.

A fundamental question arising with respect to $\alpha$-value is: which values—either among those already proposed in the literature or any new potential ones—can be defined as an $\alpha$-value? A key result of our work is that we prove the marginality approach encompasses *all values* that satisfy Shapley’s original axiomatization and exactly those.

**Theorem 2.** The value $\varphi$ can be obtained using the marginality approach if and only if it satisfies Efficiency, Symmetry, Additivity and the Null-Player Axiom.

Next, we analyze how properties of an $\alpha$-value translate into properties of weights $\alpha$. In particular, we focus on the axioms known as *Weak Monotonicity*, *Strong Monotonicity*, *Strong Symmetry*, and *Strong Null-Player*. Weak (Strong) Monotonicity states that, if we increase the value of a coalition containing a player, the payoff of this player will not decrease (will increase). We prove that $\alpha$-value satisfies Weak (Strong) Monotonicity if and only if weights $\alpha$ are non-negative (positive).

**Lemma 1.** An $\alpha$-value satisfies Weak (Strong) Monotonicity if and only if $\alpha_i(S, P) \geq 0$ ($\alpha_i(S, P) > 0$) for every significant weight.

The Strong Symmetry axiom requires that the value of any coalition has a symmetric influence not only on the payoffs of its members but also on the payoffs of all non-members. We

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\(^1\)This result comes from the paper *Steady Marginality: A Uniform Approach to Shapley Value for Games with Externalities* [21].

\(^2\)All general results concerning marginality approach (Theorems 1-5, Lemma 1, Corollary 1) comes from the paper *Reconsidering the Shapley Value in Games with Externalities* [24], which was invited for resubmission to Theoretical Economics.
prove that the $\alpha$-value satisfies Strong Symmetry if and only if weights $\alpha$ are such that the permutation in which players leave the grand coalition does not affect the probability that a given coalition structure is eventually created. We say that weights $\alpha$ satisfying this condition are **interlace resistant**.

**Theorem 3.** An $\alpha$-value satisfies Strong Symmetry if and only if $\alpha$-marginality is interlace resistant.

As a corollary to this result we have that the average approach of translating the Shapley value to games with externalities, proposed by Macho-Stadler et al. [13], is a subclass of the marginality approach and is equivalent to the marginality approach used with interlace resistant weights.

The Strong Null-Player Axiom states that a player who does not have an impact on the values of coalitions in the game does not affect the payoff division—that is, if we remove a null-player the payoffs from the game will stay the same. We prove that if $\alpha$-value satisfies Strong Symmetry than it satisfies the Strong Null-Player Axiom if and only if weights $\alpha$ are such that the probability of joining a particular coalition depends only on the other coalitions in the coalition structure and not on the coalition that is being left. This condition on weights $\alpha$ we call **expansion resistance**.

**Theorem 4.** If an $\alpha$-value satisfies Strong Symmetry then it satisfies the Strong Null-Player Axiom if and only if $\alpha$-marginality is expansion resistant.

Although the interlace and expansion resistance conditions may at first appear somewhat arbitrary, they are in fact key to understanding the relationship between the $\alpha$-parameterized Shapley axiomatization and the Myerson axiomatization based on the concept of balanced contributions extended to games with externalities. In this respect, we prove that the $\alpha$-value satisfies Myerson’s axioms (Efficiency, $\alpha$-parametrized Balanced Contributions) if and only if $\alpha$ is interlace and expansion resistant.

**Theorem 5.** Shapley’s marginality-based axiomatization (Efficiency, Symmetry, Additivity and $\alpha$-Null-Player Axiom) is equivalent to Myerson’s axiomatization (Efficiency, $\alpha$-Balanced Contributions) if and only if $\alpha$ is interlace and expansion resistant.

Our work connects with the work of Fujinaka [5], who studied Young’s axiomatization and provided a general theorem: for every definition of marginal contribution there exists a unique value which satisfies the Efficiency, Symmetry and $\alpha$-Marginality Axioms [5]. For every $\alpha$, the value proposed by Fujinaka based on Young’s axiomatization is equal to our value (derived by Theorem 1). This means that both axiomatizations are equivalent.

**Corollary 1.** Shapley’s marginality-based axiomatization (Efficiency, Symmetry, Additivity and $\alpha$-Null-Player Axiom) is equivalent to Young’s axiomatization (Efficiency, Symmetry and $\alpha$-Marginality Axiom). Moreover, both axiomatizations yield a unique value.

Finally, we present the first – to our knowledge – approximation algorithm for evaluating extended Shapley values.\(^3\) We present the general scheme of the algorithm that works for every

\(^3\)This algorithm comes from the paper The Shapley axiomatization for values in partition function games [23].
\(\alpha\)-value, thus every value obtained using marginality approach with some weighting \(\alpha\). The general scheme is based on the Monte Carlo sampling. In games without externalities the natural way is to sample over all possible permutation and for a given permutation gather players’ marginal contributions. In games with externalities we randomly select not only permutation, but also partition. Here, the probability distribution is given by the weights \(\alpha\) and strongly depends on the value that is approximated. Thus, for every existing \(\alpha\) we specify how to choose a random partition with the corresponding probability. Especially, we propose a new method to select a random partition that is needed for Hu and Yang’s value.

Part II. Algorithms for the Shapley value for graph-restricted games

While the conventional model of a coalitional game assumes that any coalition can be created and may have an arbitrary value, there are many realistic settings where this assumption does not hold. Often, agents can communicate and cooperate only via some limited number of bilateral channels. If there is no direct channel between two agents, cooperation can be still possible indirectly, through an intermediary or a sequence of them. However, when no direct or indirect connection exists between agents, they cannot coordinate their activities. Such restrictions emerge in a variety of domains including: sensor networks, telecommunications, social networks analysis, trade agreements, political alliances, etc.

An influential approach for representing such scenarios was introduced by Myerson [16], who described a coalitional game over a graph \(G = (V, E)\) in which nodes \(V\) represent agents and edges \(E\) represent communication channels between them. In graph-restricted games only connected coalitions could be assigned an arbitrary value (as the agents within are able to communicate and create value added). We say that coalition is connected if there exists a path between any two members in the graph (i.e., the subgraph induced by its members is connected). We will denote the set of all connected coalitions by \(\mathcal{C}(G)\) (or \(\mathcal{C}\) when \(G\) is known from the context). To formalise graph-restricted games, let us first consider a new value function which corresponds to \(v\) but is only defined over connected coalitions:

\[
v_G : \mathcal{C}(G) \rightarrow \mathbb{R} \quad \text{and} \quad \forall S \in \mathcal{C}(G) v_G(S) = v(S).
\]

This definition can be extended to incorporate disconnected coalitions; this has been done in two ways:

- Myerson argued that it is natural to consider a disconnected coalition as a set of disjoint, connected components. Each such component \(S'\) is, by definition, a coalition in \(\mathcal{C}(G)\) whose members are able to attain a payoff of \(v_G(S') = v(S')\). This leads to the following characteristic function, defined over both connected and disconnected coalitions [16]:

\[
v_G^M (S) = \begin{cases} v(S) & \text{if } S \in \mathcal{C}(G) \\ \sum_{K_i \in K(S)} v(K_i) & \text{otherwise,} \end{cases}
\]

where \(K(S)\) denotes the set of connected components of coalition \(S\) and \(M\) stands for Myerson. In other words, the payoff available to a disconnected coalition is the sum of payoffs of its connected components.
More recently, Amer and Gimenez [2] formalized an alternative approach to evaluate disconnected coalitions, where they assumed that all such coalitions have a value of 0. Under this assumption, they defined the following characteristic function:

$$\nu_A^G(S) = \begin{cases} 1 & \text{if } S \in C(G) \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where $A$ stands for Amer and Gimenez. The game with the above function will be called a 0-1-connectivity game. This function was later extended by Lindelauf et al. [12] to:

$$\nu_f^G(S) = \begin{cases} f(S, G) & \text{if } S \in C(G) \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where $f$ is an arbitrary function.

Since $\nu_M^G$ and $\nu_A^G$ are defined over all $2^{|V|}$ coalitions, the Shapley value can be applied as a solution concept to both of these functions. The Shapley value calculated for the first game, $SV_i(\nu_M^G)$, is called Myerson value and will be denoted $MV_i(\nu_G)$. In particular, Myerson showed that this value is the unique payoff division scheme that is efficient and rewards any two connected agents equally from the bilateral connection between them. The Shapley calculated for the second function, where only connected coalitions have non-zero value, will be simply called Shapley value for graph-restricted games.

The computation of both of these values is challenging [14, 1]. As for the Shapley value, Michalak et al. recently proposed an algorithm to compute it for an arbitrary graph-restricted game [14]. As for the Myerson value, its computational aspects were considered for certain classes of graphs and/or games. However, to date there is no algorithm for computing the Myerson value in arbitrary graphs.

**Results:** In this part of the thesis we develop two efficient algorithms, one for computing the Shapley value, and the other one for the Myerson value. Unfortunately, calculating Shapley value in general requires enumerating all $2^{|V|}$ coalitions. However, we show that traversing only connected coalition is sufficient to calculate Shapley value in graph-restricted games for both types of the characteristic function (see Theorems below). Thus, the fast enumeration of all connected coalitions is the cornerstone upon which we build both algorithms.

As in our setting, a connected coalition is a set of nodes that induces a connected subgraph, listing of all connected coalitions means enumerating all connected induced subgraphs of the graph – one of the fundamental operations in graph theory. In our thesis, we propose a new algorithm designed for this purpose.\(^4\)

Broadly speaking, our enumeration algorithm traverses the graph in a depth-first manner, and uses a divide-and-conquer technique. We start with a single node and try to expand it to a bigger connected subgraph. Whenever a new node is analyzed, we explore all its edges one by one, and when we find a new—not yet discovered—node, we split the calculations into two parts: in the first one, we add a new node to our subgraph; in the second one, we mark this node as forbidden and never enter it again. Thus, the first part enumerates subgraphs with, and the second one without, the new node.

\(^4\)Algorithm for enumeration of induced connected subgraphs and both algorithms for Myerson and Shapley value in graph-restricted games along with Lemmas 2-3 and Theorems 6-7 comes from the paper *Algorithms for the Myerson and Shapley values in graph-restricted games* [22].
To date, the state-of-the-art algorithm for enumerating connected induced subgraphs was proposed by Moerkotte and Neumann [15]. As opposed to our algorithm, which traverses the graph in a depth-first manner, their algorithm uses breadth-first search. Both algorithms run in a linear time in respect to the number of induced connected subgraphs in graph. Still, our experiments show that our new algorithm outperforms BFS enumeration two or even three times.

To support our empirical results, we provide two lemmas, which show that for cliques our algorithm performs approximately two times fewer steps (examining edges is the key component of main loops in both algorithms).

**Lemma 2.** EnumerateCSG examines edges $2^{n-1}(n^2 - 3n + 2) + (n - 1)$ times for an $n$-clique.

**Lemma 3.** DFSEnumerate examines edges $2^{n-2}(n^2 - n + 4) - (n + 1)$ times for an $n$-clique.

We also show that, unlike the state of the art, our algorithm can easily be extended to capture extra information about each enumerated subgraph, which is the crucial observation for one of our algorithms.

Building upon the above enumeration algorithm, we propose a new algorithm to compute the Shapley value for graph-restricted games. Our algorithm is based on the following theorem.

**Theorem 6.** Shapley value for game $\nu^f_G$ satisfies the following formula:

$$SV_i(\nu^f_G) = \sum_{S \in C} mc_i(S),$$

where $mc_i(S)$ stands for

$$mc_i(S) =
\begin{cases}
\xi_S f(S) & \text{if } v_i \in S \text{ and } S \setminus \{v_i\} \not\in \mathcal{C}, \\
\xi_S (f(S) - f(S \setminus \{v_i\})) & \text{if } v_i \in S \text{ and } S \setminus \{v_i\} \in \mathcal{C}, \\
-\xi_{S \cup \{v_i\}} f(S) & \text{if } v_i \not\in S \text{ and } S \cup \{v_i\} \not\in \mathcal{C}, \\
0 & \text{otherwise.}
\end{cases}$$

Based on the above observations it is crucial to not only enumerate all connected subgraphs, but also identify the cut vertices (a node whose removal disconnects a subgraph), and the neighbours, of each enumerated subgraph. As for the identification of neighbours, it can easily be done. The harder part is to identify the cut vertices. Against this background, we present the first dedicated algorithm that not only enumerates all connected subgraphs, but at the same time identifies cut vertices in each subgraph. To make this possible, our algorithm traverses all connected subgraphs in a depth-first-search (DFS) manner (as discussed above) and at the same time applies the state-of-the-art algorithm for finding cut vertices, due to Hopcroft and Tarjan [9] (also based on the DFS). Finally, we show that the algorithm for Shapley value is faster than the state of the art, due to Michalak et al. [14].

Furthermore, we propose an algorithm for the Myerson value, which is, to our knowledge, the first one in the literature for arbitrary graph. Here, algorithm is based on the following theorem.

**Theorem 7.** The Myerson value for graph-restricted games satisfies the following formula:

$$MV_i(\nu_G) = \sum_{S \in \mathcal{E}, v_i \not\in S} \xi_1 v(S) - \sum_{S \in \mathcal{E}, v_i \not\in S} \xi_2 v(S),$$

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where
\[ \zeta_1 = \frac{(|S| - 1)!|\mathcal{N}(S)|!}{(|S| + |\mathcal{N}(S)|)!}, \quad \zeta_2 = \frac{|S|!(|\mathcal{N}(S)| - 1)!}{(|S| + |\mathcal{N}(S)|)!}, \]
and \( \mathcal{N}(S) \) denotes the set of neighbours of coalition \( S \).

In this case, our algorithm enumerates all connected coalitions and for all members and neighbours update the Myerson value according to the above formula.

We test both algorithms on an interesting application. There is currently much interest in the possibility of applying social network analysis techniques to investigate terrorist organizations [19]. A particular attention is paid to the problem of identifying key terrorists. This not only helps to understand the hierarchy within these organizations but also allows for a more efficient deployment of scarce investigation resources [11]. One possible approach to this problem is to try and infer the importance of different individuals from the topology of the terrorist network. In graph theory, such an inference can be obtained in various ways, depending on the adopted centrality measures, i.e., the adopted way to measure the centrality, i.e., importance, of different nodes in a network, based on its topology. A number of researchers have proposed to incorporate game-theoretic techniques into existing centrality measures [8, 6]. Especially, Lindelauf et al. [12] tried to develop a centrality measure that assesses the role played by individual terrorists in a way that accounts for the following two factors: the terrorists’ role in connecting the network and additional intelligence available about the terrorists. To this end, Lindelauf et al. proposed to use the Shapley value for graph-restricted games (according to the definition of \( v^f_G \)).

Our dedicated algorithm allows us to perform a sensitivity analysis of Lindelauf et al.’s centrality. Our results suggest that for sparse networks – and terrorist networks tend to be sparse [11] – the connectivity factor is over-represented in the measure based on \( SV_i(v^f_G) \). As a result, we claim that the additional intelligence factor hardly ever affects the ranking. Given this, we propose to use Myerson value instead of the Shapley value as a centrality measure and show that it is more suitable for this particular application.

Finally, we address the problem of approximation of Shapley value for graph-restricted games. In many settings even the fastest exact algorithms fail to return the result in a reasonable amount of time. To this end, we propose an approximation algorithm for Shapley value in graph-restricted games and for two more complex definitions of a game used in the gatekeepers metric.\(^5\)

\(^5\)Approximation algorithm for Shapley value in graph-restricted games comes from the paper \textit{Computational Analysis of Connectivity Games with Applications to the Investigation of Terrorist Networks} [14]. Approximation algorithm for gatekeepers metric comes from the paper \textit{A Shapley value-based approach to determine gatekeepers in social networks with applications} [18].

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