

# Perturbative Corrections to the Inclusive Radiative $B$ Meson Decay

PhD thesis

Abstract

The main subject of my Ph.D. thesis is the calculation of the unknown corrections to the radiative  $b$  quark decay  $b \rightarrow X_s^p \gamma$  which at the hadronic level corresponds to the radiative  $B$  meson decay  $\bar{B} \rightarrow X_s \gamma$ . Perturbatively defined decay rate  $\Gamma(b \rightarrow X_s^p \gamma)$  approximately equals to the  $\bar{B} \rightarrow X_s \gamma$  decay rate.

Precise knowledge of the decay  $\bar{B} \rightarrow X_s \gamma$  within the Standard Model provides powerful constraints on the theories beyond Standard Model. Two Higgs Doublet Models, supersymmetric theories or models with extra spacetime dimensions are the most significant examples.

The Ph.D. thesis includes two calculations. First, we found tree-level contributions from CKM-suppressed  $b \rightarrow u\bar{u}s\gamma$  transitions together with similar ones from the QCD penguin operators. Weak radiative decay  $\bar{B} \rightarrow X_s \gamma$  is known to be a loop-generated process. However, it does receive the aforementioned tree-level contributions. For a low value of the photon energy cutoff  $E_0 \sim m_b/20$  that has often been used in the literature, they can enhance the inclusive branching ratio by more than 10%. For  $E_0 = 1.6$  GeV or higher, the effect does not exceed 0.4%, which is due to phase-space suppression and smallness of the relevant Wilson coefficients. This tiny numerical value does not exceed previous rough estimations and justifies why these contributions have been neglected up till now. The perturbative results contain collinear logarithms that depend on the light quark masses  $m_q$  ( $q = u, d, s$ ). We have allowed  $m_b/m_q$  to vary from 10 to 50, which corresponds to values of  $m_q$  that are typical for the constituent quark masses.

Second part of the Ph.D. thesis is devoted to calculation of the perturbative  $\mathcal{O}(\alpha_s^2)$  corrections to the branching ratio  $\text{BR}(B \rightarrow X_s \gamma)$  in the Brodsky-Lepage-Mackenzie approximation. They receive contributions from two-, three- and four-body final states. While all the two-body results are well established by now, the other ones have remained incomplete for several years. We have calculated the last contribution that has been missing

to date, namely the one originating from interference of the current-current and gluonic dipole operators ( $K_{18}^{(2)\beta_0}$  and  $K_{28}^{(2)\beta_0}$ ). Moreover, we confirmed all the previously known results for BLM corrections to the photon energy spectrum that involve the current-current operators (e.g.,  $K_{22}^{(2)\beta_0}$  and  $K_{27}^{(2)\beta_0}$ ). Finally, we also confirm the findings of Ferroglia and Haisch on self-interference of the gluonic dipole operator ( $K_{88}^{(2)\beta_0}$ ) which was published in the same time.

Most of the found corrections do not exceed  $\pm 1\%$  of the branching ratio central value. The only exception is  $K_{22}^{(2)\beta_0}$  correction which affects the branching ratio by  $+1.9\%$ , which still remains within the assumed  $\pm 3\%$  uncertainty for all such effects. In spite of the smallness of the found corrections, their knowledge is crucial for reducing the theory uncertainty in the close future.

The thesis is supplemented with four appendices. Appendix A includes description of the integration over four-particle phase space without neglecting light quark masses. Particular attention is paid to the extraction of the collinear logarithms with the possibility of staying differential in the photon energy. Appendix B is devoted to the derivation of the splitting functions in the light-cone axial gauge. Splitting functions describe photon behaviour at small angles relative to the emitter. They are commonly used to extract collinear logarithms in processes with radiation. In the considered calculation however, they are used in order to compare the results received in dimensional regularisation with those where the collinear emission is regulated by the light quark masses. Such a comparison is important to perform the crosscheck of the results. Appendix C includes description of Smith-Voloshin method which was used in the calculation of the NNLO corrections in the BLM approximation. Appendix D contains analytic functions necessary in the evaluation of the  $K_{22}^{(2)\beta_0}$  correction.