Electrod-positron pair creation, electric conductivity in graphene and the classical limit of the Dirac operator in terms of the Dirac-Heisenberg-Wigner function

A PHD dissertation performed in Center for Theoretical Physics PAS under the supervision of prof. dr. hab. Iwo Białynicki-Birula and submitted to the Faculty of Physics at the University of Warsaw.

Abstract

In my PHD dissertation I used an approach of the one-time Dirac-Heisenberg-Wigner (DHW) function to investigate three problems which are present in the title of the thesis, i.e.: electron-positron pair creation in homogeneous electric field, electronic transport in graphene and the classical limit of the second-quantized Dirac equation.

In the first part I derived the density of electron-positron pairs in terms the elements of the DHW matrix. Using properties of the groups $SO(3)$ and $SU(2)$ I simplified equations satisfied by the DHW functions in the case of the homogeneous electric field to the form of a single, ordinary, second order differential equation. I solved that equation in cases of a constant electric field, the electric field switched on exponentially and for the Sauter field. In all cases I discussed the analiticity of the density of created pairs. Moreover, I showed that when the field is switched on adiabatically, the density of created pairs is an analytic function of the electric field strength. In the case of the constant electric field $E$ I derived first terms of the power series expansion of $n(E)$ with respect to $E$ and I presented a perturbative method to generate the total power series expansion. I shown that the Schwinger nonanalyticity appears in the unphysical limit, when the electric field acts on the system infinitely long. Thus, obtained results suggest that the nonanalytic character of the Schwinger formula $n(E) = e^{-\pi E/E_{\text{crit}}}$, where $E_{\text{crit}} = m^2 c^3 / (e \hbar)$ denotes the „critical field strength”, is implied by the unphysical assumption about the interaction of the electric field with the Dirac vacuum.
In the second part I performed a detailed discussion of a connection between the Dirac equation and graphene. I introduced the DHW function for graphene and presented a general formula for the carrier current in terms of the DHW functions. Solving the graphene DHW equations in the case of crossed electric and magnetic fields I rederived the predictions of the quantum Hall effect in graphene. I showed that using the formalism of the DHW function one can include additional information about the system investigated, like the localizability of the carrier states. Finally, I presented a perturbative method which leads to corrections of the formula for the Hall current. I examined an utility of the presented method, in particular I pointed out all its drawbacks.

In the last part I discussed the classical limit of the Dirac operator, i.e. the classical limit of its expectation value taken on an arbitrary state. Using the method of the DHW function I presented a decomposition of the spin density into its particle and antiparticle contributions.