Terahertz magnetospectroscopy of high electron mobility CdTe/CdMgTe quantum wells

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Thesis submitted for
the Doctor of Philosophy
(PhD) degree

Warsaw
May, 2015
PREFACE

Bulk CdTe is a semiconductor known already for more than 130 years, since it was first synthesized in 1879 by Margottet [1]. Such properties as direct bandgap of 1.52 eV, high absorption coefficient, the feasibility of $p$- and $n$-type doping and relatively low cost of thin film technology, make CdTe one of the most promising materials for the photovoltaics (PV) [2–9]. Intensive investigation of CdTe-based PV modules has developed to an industrial production [10].

Due to a large atomic number, crystals of CdTe also saw the wide applications as detectors for a high energy radiation: X-ray [11–16] and $\gamma$-ray [17–23].

On the other hand, a large Fröhlich constant ($\alpha = 0.3$) and the energies of optical phonons falling in the far-infrared region of electromagnetic spectrum, make CdTe an interesting subject for a far-infrared (THz) spectroscopy. An extensive THz studies were performed on a bulk CdTe which revealed such fundamental phenomena as, for example, the polaron cyclotron resonance [24], the interaction between a phonon and a free carrier plasma [25] or magnetopolaron effect in shallow impurity transitions [26].

In a low dimensional solid state physics, the field of spintronics in CdTe quantum dots with magnetic ions is emerging rapidly [27–30]. It is led by the idea to develop a solid-state light controlled magnetic memory cells. However, when it comes to CdTe-based quantum wells (QW), they are not so extensively studied, especially in comparison with a III-V semiconductor structures, mainly due to a technological limitations resulting to a low mobility of a 2DEG.

The current progress in molecular beam epitaxy (MBE) growth of CdTe-based systems of a low dimensionality allowed to carry out a number of advanced experiments on a high quality CdTe/CdMgTe and CdMnTe/CdMgTe QWs. A resistance quantization in a quantum point contact (QPC) [31], a fractional quantum Hall effect [32, 33] as well as spin current effects [34, 35] were studied. An excitation of THz radiation induced by optically generated spin waves in CdMnTe/CdMgTe containing a high mobility 2D electron plasma was demonstrated [36].

However, there are only a few reports on a THz spectroscopy in these systems. The cyclotron resonance was investigated, the electron cyclotron mass was determined and a polaron effect was observed in modulation doped CdTe/CdMgTe quantum wells of different
The electron cyclotron mass was also determined in nominally undoped CdTe/CdMnTe quantum wells of different width using the optically detected cyclotron resonance technique [40] and a good agreement with the results on the electron cyclotron mass presented in Ref. [39] was found. Collective excitations of 2D magnetoplasma were reported in the Raman spectroscopy experiment on modulation doped CdTe/CdMnTe QWs [41]. Also, an impurity shifted cyclotron resonance was reported in Ref. [37] and shallow impurity transitions in CdTe/CdMgTe quantum wells continuously doped with iodine were studied [42, 43].

The main goal of this work was to study a high mobility 2DEG in a single asymmetrically modulation doped CdTe/CdMgTe QW by the means of THz magnetospectroscopy. The experiments were carried out at liquid helium temperatures and in quantizing magnetic fields on the samples of different geometry (two-contact, grid-gated, QPC and Hall bar).

We have observed a variety of physical phenomena such as cyclotron resonance, Shubnikov-de Haas oscillations in photocurrent, the THz radiation induced breaking of quantum Hall state. Also, in a grid-gated samples 2D plasmons were observed and their dispersion was found to be influenced by an interaction with a crystal lattice vibrations. To the best of our knowledge, this is the first report on 2D plasmons observed in CdTe-based QW in a far-infrared experiment.

Also, we have investigated a THz excited photocurrent in a QPC in a magnetic field. In the literature only a few papers which consider this topic can be found. The THz excited photocurrent was reported on GaAs/AlGaAs and InAs/InAlAs QPCs as a function of the gate voltage and magnetic field. For the interpretation of the experimental results two models were proposed: a bolometric heating of a 2DEG [44, 45] and a high frequency rectification due to a nonlinear conductance of the quantum point structure [46–48]. Although, in our experiments we have found that the features observed in photocurrent result mainly from a 2D plasmon excitation.

The structure of thesis is as follows. Part 1 introduces briefly the main theoretical concepts which help to understand the results presented in this work. In Part 2 the samples and experimental part of the work is discussed. An experimental results on magnetotransport are presented and discussed in Part 3. Polaron effect experimental results and theoretical treatment are shown in Part 4. Part 5 is devoted to discuss the THz induced photocurrent in the investigated samples, the mechanisms of photocurrent generation and peculiarities of
the spectra in magnetic field. In Part 6 we present the results on magnetoplasmons with a detailed theoretical treatment. We also point out there the importance of plasmon-phonon interaction in the 2D plasmon dispersion. Finally, in Part 7 the resonant detection (CR peak magnitude) is investigated as a function of electron concentration. At the end we summarize the main results of the work.
AKNOWLEDGEMENTS

At first I would like to express my huge gratitude to the thesis Supervisor dr hab. Jerzy Łusakowski prof. (UW) for a possibility to spend an interesting and fruitful time with his group in Warsaw. I admire very much his broad knowledge of physics and a wide spectrum of interests. Also, I am grateful for the patience, never ending optimism, strong support and a non-formal attitude to my problems inside and outside the laboratory.

Also, I am indebted to prof. dr hab. Marian Grynberg who has shared with me his unbelievably deep knowledge and experience in solid state physics as well as in life in general.

I want to thank to prof. Wojciech Knap for kindness, enthusiasm and the help on various problems not necessarily connected to science during my outgoing visits to Montpellier, France.

My gratitude also goes to dr. Karol Nogajewski, dr. Krzysztof Karpierz and Mr. Marcin Bialek – Colleagues from the far-infrared transistor laboratory at the University of Warsaw, who introduced me to the brisk life of the lab and helped to overcome challenges I encountered while obtaining and interpreting the experimental results.

I am grateful to people from the Charles Coulomb Laboratory at Montpellier University 2: dr. Nina Dyakonova, dr. Dmytro But, dr. Christophe Chaubet, dr. Dominique Coquillat, dr. Frederic Teppe and dr. Abdel El Fatimy who had helped me a lot with the experiments and had created a diligent and at the same time relaxed atmosphere in the lab.

Last but not least, I thank to my Family for their fantastic support and the respect to my chosen way. Also, I am grateful to all of my Friends who did not let me get bored during my spare time, be it Warsaw, Montpellier or Vilnius.

Also, the support from the Foundation for Polish Science through the International PhD Projects Program co-financed by the EU European Regional Development Fund is acknowledged.
The following publications resulted from the preparation of this PhD thesis:


The results of the investigation discussed in this PhD thesis were presented in the following conferences:

1. Posters:
(a) I. Grigelionis, M. Grynberg, Z. Adamus, G. Karczewski, T. Wojtowicz and J. Łusakowski, “Terahertz detectors based on a gated two-dimensional electron plasma in CdMnTe/CdMgTe quantum wells”, 39th International Conference on Infrared, Millimeter, and Terahertz waves (IRMMW-THz), September 14 - 19, 2014, Tucson, AZ, USA;


(c) I. Grigelionis, K.Nogajewski, M. Białek, K. Karpierz, M. Grynberg, T. Wojtowicz, G. Karczewski, and J. Łusakowski, “High electron mobility plasma in CdTe/CdMgTe quantum wells for magnetic-field tunable detection of THz radiation”, 31st ICPS, July 29 - August 3, 2012, Zurich, Switzerland;


2. Oral talks:


Part I

Theoretical Overview

I. CARRIER MOTION IN TWO DIMENSIONS IN THE MAGNETIC FIELD

In order to describe a 2DEG energy spectrum in an external magnetic field $B = (0, 0, B_z)$ perpendicular to the 2DEG sheet, the following Schrödinger equation must be solved for the 2D electrons neglecting the spin and an electron-electron interaction:

$$\left[ \frac{1}{2m_e^*} \left( -i\hbar \nabla + eA \right)^2 + V(z) \right] \Psi = E \Psi,$$  \hspace{1cm} (I.1)

where $m_e^*$ is the electron effective mass, $A = \text{rot}B$ is the vector potential of magnetic field and $V(z)$ is the confining potential of the quantum well. In a Landau gauge, $A = (0, B_x, 0)$, Eq. (I.1) is

$$\left[ -\hbar^2 \frac{\partial^2}{2m_e^* \partial x^2} + \frac{1}{2m_e^*} \left( -i\hbar \frac{\partial^2}{\partial y^2} + eBx \right)^2 + \frac{-\hbar^2}{2m_e^*} \frac{\partial^2}{\partial z^2} + V(z) \right] \Psi = E \Psi.$$  \hspace{1cm} (I.2)

Now we can apply the method of variables separation, what allows us to write the solution of the Schrödinger equation as a product of partial wavefunctions:

$$\Psi = Ce^{ik_y y}\varphi(x), E = \epsilon_{||} + \epsilon_{\perp}.$$  \hspace{1cm} (I.3)

For an electron motion in a $xy$-plane, the wavefunction $\varphi(z)$ and “perpendicular” energy $\epsilon_{\perp}$ are neglected and Eq. (I.2) for the energy $\epsilon_{||}$ can be expressed as

$$\left[ -\hbar^2 \frac{d^2}{2m_e^* dx^2} + \frac{m_e^* \omega_c^2}{2} (x - x_0)^2 \right] \varphi = \epsilon_{||} \varphi,$$  \hspace{1cm} (I.4)

with the cyclotron frequency $\omega_c = eB/m_e^*$ and parameter $x_0 = \hbar k_y/eB$. The latter equation is the equation for the harmonic oscillator with a frequency of oscillations equal to $\omega_c$ and the equilibrium point $x_0$. Therefore, the energy spectrum and wavefunctions are:

$$\epsilon_j = \hbar \omega_c \left( j + \frac{1}{2} \right), j = 0, 1, 2, ..., \hspace{1cm} (I.5)$$

$$\varphi_j = \frac{1}{\sqrt{l_B}} \frac{1}{\sqrt{2^j j! \sqrt{\pi}}} \exp \left( -\frac{u^2}{2} \right) H_j(u), u = \frac{x - x_0}{l_B}, \hspace{1cm} (I.6)$$

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where $H_j(u)$ is a Hermite polynomial of the order $j$ and $l_B$ is the so-called magnetic length,

$$l_B = \sqrt{\frac{\hbar}{eB}}.$$  \hfill (I.7)

Thus, the energy spectrum in 2D consists of discrete levels, known as Landau levels (LLs), separated by the energy gap of $\hbar\omega_c$.

Due to the spin, an electron has its own magnetic moment which interacts with the external magnetic field. The energy of interaction is

$$V_s = g\mu_B sB,$$  \hfill (I.8)

where $g$ is a Landé factor, $\mu_B = e\hbar/2m_0$ ($m_0$ – free electron mass) stands for the Bohr’s magneton and $s$ is the spin operator. Including the spin interaction with the magnetic field in Eq. (I.1) results in a splitting of LLs to separate branches where the spin is oriented up or down. The energies of these levels would shift from a degenerate level (when the spin interaction is neglected) energy by:

$$\Delta \epsilon_Z = \pm \frac{g}{4} \frac{m}{m_0} \hbar \omega_c,$$  \hfill (I.9)

for the spins $\uparrow$ and $\downarrow$, respectively.

II. DENSITY OF STATES IN TWO DIMENSIONS

A density of states in a 2DEG can be calculated considering the general formula (the spin is neglected),

$$D(\epsilon) = \frac{1}{4\pi^2} \int \delta[\epsilon - \epsilon(k_\perp)]dk_\perp.$$  \hfill (II.1)

When $B = 0$, in a QWs with a parabolic dispersion the step-like density of states is equal to:

$$D(\epsilon) = \frac{m}{\pi\hbar^2} \sum_{n=0}^{\infty} \vartheta(\epsilon - \epsilon_n),$$  \hfill (II.2)

where $\vartheta$ is a unit step-like function and summation is done over all energy subbands in a QW. In a perpendicular magnetic field the density of states is:
\[ D(\epsilon, B) = \frac{eB}{\hbar} \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \delta \left( \epsilon - \epsilon_n - \left( j + \frac{1}{2} \right) \hbar \omega_c \right). \] (II.3)

III. MAGNETOTRANSPORT IN A TWO-DIMENSIONAL ELECTRON GAS

A. Classical magnetotransport

Here we will describe the magnetotransport within the frame of a simple Drude model including the scattering of carriers through momentum relaxation time (transport scattering time) \( \tau_m \). At zero magnetic field and in a constant electric field \( E \), electrons close to the Fermi level are directed to move with a drift velocity \( v_d \). The kinetic equation is:

\[
\left( \frac{dp}{dt} \right)_{\text{sat}} = \left( \frac{dp}{dt} \right)_{\text{ext}}, \quad (\text{III.1})
\]

that is, the momentum of electrons gained from the external electric field is lost at the same rate by scattering events. Then, the drift velocity is equal to:

\[
v_d = \frac{e\tau_m}{m^*} E = \mu E. \quad (\text{III.2})
\]

Here, the quantity \( \mu = e\tau_m/m_e^* \) is called the electron mobility and it can be found from the Hall effect experiment. The electron current density \( j \) with an electron concentration \( n_s \) is:

\[
j = n_s e v_d = n_s e^2 \tau_m / m_e^* E. \quad (\text{III.3})
\]

This equation represents the Ohm’s law. Conductivity is defined as \( \sigma_0 = n_s e^2 \tau_m / m_e^* \), while the resistivity is \( \rho_0 = 1/\sigma_0 \). When the mobility is known, the \( \tau_m \) is also easy to find. In an external magnetic field, perpendicular to \( E \)-field, electrons move on curved trajectories, influenced by the Lorentz force. In such a case, the Eq. (III.2) is modified as follows:

\[
\frac{v_d m_e^*}{\tau_m} = e \left( E + v_d \times B \right), \quad (\text{III.4})
\]

with \( B \) representing the magnetic field vector. In general, the conductivity is a tensor quantity \( \hat{\sigma} \), thus Ohm’s law for 2D electrons in magnetic field is expressed as:
\[
\begin{pmatrix}
  j_x \\
  j_y
\end{pmatrix} = 
\begin{bmatrix}
  \sigma_{xx} & \sigma_{xy} \\
  \sigma_{yx} & \sigma_{yy}
\end{bmatrix}
\begin{pmatrix}
  E_x \\
  E_y
\end{pmatrix};
\]

\[
\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1 + \omega_c^2 \tau_m^2};
\]

\[
\sigma_{xy} = -\sigma_{yx} = -\frac{\sigma_0 \omega_c \tau_m}{1 + \omega_c^2 \tau_m^2}.
\]

Here \( \omega_c = eB/m_e^* \) is the cyclotron frequency for the electron orbiting in a magnetic field. Because \( \hat{\rho} = \hat{\sigma}^{-1} \), the components of the resistivity tensor can be found by taking the inverse matrix of the conductivity tensor in Eq. (III.5). After inverting \( \hat{\sigma} \), we find:

\[
\hat{\rho} = 
\begin{bmatrix}
  \rho_{xx} & \rho_{xy} \\
  \rho_{yx} & \rho_{yy}
\end{bmatrix} = \left( \sigma_{xx}^2 + \sigma_{xy}^2 \right)^{-1}
\begin{bmatrix}
  \sigma_{yy} & -\sigma_{yx} \\
  -\sigma_{xy} & \sigma_{xx}
\end{bmatrix}.
\]

During the Hall effect measurements, the terms of the tensor \( \hat{\rho} \) are directly measured. Using Eqs. (III.6), (III.7) and (III.8) we find \( \rho_{xx} \) and \( \rho_{xy} \):

\[
\rho_{xx} = \rho_{yy} = \frac{m}{e^2 \eta_0 \tau_m} = \frac{E_x}{j}, \quad \rho_{xy} = -\rho_{yx} = \frac{B}{en_0} = R_H.
\]

Here \( R_H \) denotes the Hall resistance. A longitudinal resistance does not depend on the magnetic field, while the Hall resistance increases linearly with \( B \).

**B. A Shubnikov-de Haas effect**

However, the magnetoconductivity model discussed above does not describe the transport in strong magnetic fields. The oscillations of the longitudinal resistance \( \rho_{xx} \) occur which are periodic in an inverse magnetic field and are referred to as Shubnikov-de Haas (SdH) oscillations. This is not a unique effect of 2D systems and it is also observed in bulk materials.

The SdH oscillations emerge because a step-like density of states of a 2DEG, constant at zero magnetic field, in a non-zero magnetic field splits to the discrete energy levels represented ideally by \( \delta \)-functions and separated by the gaps of \( h\omega_c \) (Landau quantization):

\[
D(\epsilon, B) = \frac{eB}{\hbar} \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \delta \left[ \epsilon - \epsilon_n - \left( j + \frac{1}{2} \right) \hbar \omega_c \right].
\]

Here \( \epsilon_n \) is the energy of the free electron system at \( B = 0 \) T. The spectrum of the energy levels resembles that of a harmonic oscillator. Also, it is important to remember that here
we are talking about energy levels which are not spin resolved. However, the spin splitting is important in the case of a CdTe as it shows a large Landé factor for electrons $g \approx -1.6$. The magnetic field allows to tune the energy of Landau levels as well as their degeneracy. Scanning with $B$ changes the number of occupied Landau levels, and dividing 2D electron concentration $n_s$ by $eB/h$ (degeneracy of each LL) lets us find the filling factor $\nu$, i.e. a number telling how many Landau levels are occupied:

$$\nu = \frac{n_s h}{eB}. \quad (\text{III.11})$$

When the filling factor is an integer number, the Fermi level is between two adjacent Landau levels, what gives the maximum in resistance. If the filling factor is an odd number of the halves, Fermi level is in the middle of the Landau level and the resistance is at its minimum. That is how the oscillations in resistance develops.

Now, if $\nu_1$ and $\nu_2$ are the filling factors such that $\nu_2 - \nu_1 = 1$ (for a spin degenerate case), the $n_s$ can be found from Eq. (III.11):

$$n_s = \frac{2e}{h(1/B_2 - 1/B_1)}, \quad (\text{III.12})$$

where $B_1$ and $B_2$ are magnetic field inductions at $\nu_1$ and $\nu_2$, respectively.

C. An Integer Quantum Hall Effect

In strong quantizing magnetic fields and low temperatures, a linear Hall resistance dependence on $B$ is replaced by a quantized step-like behaviour, with well developed plateaus in the vicinity of an integer $\nu$. At the same time, the longitudinal resistance drops to zero at plateaus and shows a sharp increase in a regions between plateaus. This effect was first observed by Klaus von Klitzing et al. in a Si MOS structure [49] and a few years later it was demonstrated in 2DEG in AlGaAs/GaAs heterojunction[50]. It was found that plateaus in the Hall resistance do not depend on the material and are defined by universal constants in a following way:

$$R_H = \frac{h}{e^2 j}, \, j = 1, 2, 3, ... \quad (\text{III.13})$$

Carrier transport in the QH regime is described by a conductivity tensor:
\[
\hat{\sigma} = \begin{bmatrix}
0 & e^2 j / h \\
-e^2 j / h & 0
\end{bmatrix}.
\] (III.14)

When the Fermi level is in the gap between two Landau levels, the electron scattering from a fully occupied lower LL to an empty upper LL is not possible due to a low temperature. At this condition \( \tau_m \to \infty \) and according to Eq. (III.6) \( \sigma_{xx} \) (and \( \sigma_{yy} \)) \( \to 0 \). However, using Eq. (III.8) we can show, that \( \rho_{xx} \to 0 \) as well. The longitudinal conductivity is zero, because \( j_x = 0 \) and \( j \perp E \) in strong magnetic fields. Also, there is no potential drop in a direction of the current flow, therefore \( \rho_{xx} = 0 \).

Plateaus are well explained by system imperfections, which result in a broadening of LLs. Extended electron states exist only in a narrow region around centers of broadened LLs and the states on the shoulders are localized [Fig. (III.2)]. If the Fermi level lies in a region of localized states, all extended states with lower energy are fully occupied, thus no scattering is possible, \( \rho_{xx} = 0 \) and \( \rho_{xy} \) value is finite. Increasing (decreasing) the magnetic field would put the Fermi level in a region of extended states, where scattering occurs and the Hall resistivity gradually increases to the next plateau value. The Hall resistance stays constant in plateaus to a very high accuracy, despite of imperfections of the sample. With the extended states fully occupied, the electron transport is allowed only in a direction perpendicular to \( E \) with a drift velocity \( v_{d,y} = -e E_{H} / B \). Increasing the magnetic field would result in an increase of degeneracy of the LL, but \( v_{d,y} \) would decrease at the same time, holding the Hall current

Figure III.1. Quantum Hall effect in Si MOS. Taken form [49].
constant – therefore $R_H$ does not change in the plateau region. The width of the plateau is defined by a density of imperfections in the crystal. The drift velocity of the electrons in the extended states increases to compensate for the contribution to $j_H$ coming from the electrons trapped at the localized states by the potential fluctuations.

D. A Fractional Quantum Hall Effect

In samples of a very high electron mobility the plateaus in the Hall resistance develop at the fractional values of $\nu$. That is how a fractional quantum Hall effect manifests. The first observation of FQHE at fraction $\nu = 1/3$ was made at GaAs/AlGaAs heterostructure of a very high quality, by Tsui, Stormer and Grossard [50], a few years after the IQHE was reported. Later on, much more fractions were discovered.

This phenomenon can be explained using the model of a spin resolved composite Fermions (CF) [52–54]. The main aspects of a theory needed to understand the FQHE within a frame of CF theory will be introduced briefly.

A composite Fermion is an electron which captures not the only unit flux $\Phi_0 = h/e$, but rather the even number of it. The total magnetic flux density is equal to the magnetic induction $B$, applied to a 2D electron layer. If the electron concentration is $N$, it is supposed that due to Coulomb interaction, each electron is associated with a $2p$ ($p$ being non-negative integer) number of fluxes and forms the CF, while the remaining fluxes contribute to an effective field of CF. If a number of unit flux in $B$ is defined as $q$, then $B = q\Phi_0$. For a filling factor $\nu = 1/2l$, where $l = 1, 2, 3, \ldots$, the number of unit flux $q_0 = 2lN$ and the flux

![Figure III.2. The broadened Landau levels in a 2DEG in a magnetic field. Taken from [51].](image)
density $B_0 = q_0 h/e$ are defined. In this situation, FQHE can be seen as a IQHE of composite Fermions in an effective field $B^* = |B - B_0|$, and with plateaus in the Hall resistance occurring at filling factors:

$$\nu = \frac{p}{2pl \pm 1}. \quad (\text{III.15})$$

According to this equation, around filling factor $\nu = 1/2$ ($l = 1$) a FQHE states form at filling factors $1/3$, $2/5$, $2/3$, $3/7$, $3/5$ and so on. Also, it was found for a GaAs that the effective mass of CF is around ten times an effective mass of an electron.

![Figure III.3. Magnetoresistance around $\nu = 3/2$ Landau level filling factor at various tilt angles $\theta$ as a function of normal field (taken from [55]).](image)

However, Eq. (III.15) fails to explain the FQHE states around $\nu = 3/2$, which consists of the two spin resolved LLs, filled completely with a down spin and half-filled with an up spin. In order to describe the experimentally observed Hall resistance plateaus, CF must be treated as a particle carrying a spin [55, 56]. Then the filling factors at which FQH states are expected, are given by:

$$\nu = \frac{3p \pm 2}{2p \pm 1}.$$

Away from $\nu = 3/2$ CFs are quantized to LLs by an effective field $B^* = B_\perp - B_0$, where normal field $B_\perp = B \cos \theta$ with $\theta$ as a tilt angle between the field direction and sample.
surface normal. When magnetic field is tilted at certain angles, the spin level position of one CF Landau level coincides with a spin level position of another CF Landau level and the energy gap disappears. This leads to an increased longitudinal resistivity. The evolution of FQH states with a tilt angle of magnetic field is shown in Fig. (III.3) for a 2DEG in GaAs/AlGaAs QW.

IV. RESONANT POLARON EFFECT IN TWO-DIMENSIONS

In polar semiconductors, an electron interaction with longitudinal optical lattice vibrations (LO phonons) is relatively strong. Due to this fact a free electron traveling through the crystal polarizes the crystal lattice. Since the polarization follows the electron movement, the electron gains in mass and is seen as a quasiparticle – a polaron (for an overview, see [57, 58]). This specific polaron is assumed to be large and is known as the Fröhlich polaron. The strength of the electron-phonon interaction is described by a dimensionless parameter, the Fröhlich constant [59]:

$$\alpha = \frac{e^2}{\hbar} \sqrt{\frac{m_b}{2\hbar \omega_{LO}}} \left( \frac{1}{\varepsilon_{\infty}} - \frac{1}{\varepsilon} \right),$$

where $m_b$ is the electron band mass, $\omega_{LO}$ – the LO-phonon frequency, $\varepsilon_{\infty}$ and $\varepsilon$ are a high frequency and a static dielectric constants of the material, respectively.

In an external magnetic field it is possible to achieve a condition for the resonant polaron effect $\omega_c \approx \omega_{LO}$. Also, when $\omega_c$ approaches $\omega_{LO}$ the effective mass of the electron increases due to the fact that moving electron deforms the crystal lattice, what gives rise to the change of a LL slope and anticrossing of the LLs with a LO phonon energy level $\hbar \omega_{LO}$. The cyclotron resonance spectra in magnetic field under an influence of the polaron effect can be calculated using the memory function approach [60–63]. We will follow the overview of this method adapted to two dimensions given in Ref. [64].

Motion of the electron in a polar crystal lattice in the magnetic field $\mathbf{B} = (0, 0, B_z)$ perpendicular to the 2D electron layer is described by a Fröhlich Hamiltonian [59],

$$H_F = \frac{1}{2m_b} (-i\hbar \nabla + e\mathbf{A})^2 + \sum_k \hbar \omega_{LO} a_k^\dagger a_k + \sum_k (V_k a_k e^{ikr} + V_k a_k e^{-ikr}),$$

where $a_k^\dagger$ and $a_k$ are creation and annihilation operators, respectively of a bulk LO phonon
with wavevector \( k \) and energy \( \hbar \omega_{\text{LO}} \). The potential \( V_k \) is

\[
V_k = i\hbar \omega_{\text{LO}} \left( \frac{4\pi\alpha}{Vk^2} \right) \left( \frac{\hbar}{2m_b\omega_{\text{LO}}} \right) \langle \psi_0 | e^{ikz} | \psi_0 \rangle.
\]

(IV.3)

Here, \( \psi_0(z) = \sqrt{\frac{b}{2}} e^{-bz/2} \) is the electron wavefunction in a direction normal to the 2D electron sheet, with \( b = \frac{48\pi N m_e e^2}{\hbar \varepsilon_0} \), where \( N = n_d + \frac{11}{32} n_e \) (\( n_d \) - depletion and \( n_e \) electron densities, respectively). Equation IV.3 is derived when only the first energy subband in the quantum well is taken into account.

The dynamical conductivity in linear-response theory is expressed:

\[
\sigma(\omega) = \frac{in_e e^2}{m_b (\omega - \omega_c - \sum(\omega))},
\]

(IV.4)

with a memory function \( \sum(\omega) = \sum(\alpha, \omega_c, b; \omega) \) and \( \omega_c \) as an unperturbed cyclotron frequency. The real part of Eq. IV.4 defines the magneto-optical absorption,

\[
-\text{Im} \sum(\omega) \frac{\omega - \omega_c - \sum(\omega)^2 + [\text{Im} \sum(\omega)]^2}{\omega - \omega_c - \sum(\omega)}.
\]

(IV.5)

The real and imaginary parts of the memory function are:

\[
\text{Re} \sum(\omega) = \sum_{j=0}^{\infty} \frac{B_j}{\omega \Gamma} \left[ 2D \left( \frac{\epsilon_j}{\Gamma} \right) - D \left( \frac{\epsilon_j + \omega}{\Gamma} \right) - D \left( \frac{\epsilon_j - \omega}{\Gamma} \right) \right],
\]

(IV.6)

\[
\text{Im} \sum(\omega) = \sum_{j=0}^{\infty} \frac{\sqrt{\pi} B_j}{2\omega \Gamma} \left[ \exp \left( -\frac{(\epsilon_j + \omega)^2}{\Gamma^2} \right) - \exp \left( -\frac{(\epsilon_j - \omega)^2}{\Gamma^2} \right) \right],
\]

(IV.7)

where \( \Gamma \) is a Landau level broadening, \( \epsilon_j = \omega_{\text{LO}} + j\omega_c \) (\( j = 0, 1, 2, 3, ... \)) and \( D(t) = \int_0^t e^x dx \) is a Dawson integral. The factor \( B_j \) is defined as follows:

\[
B_j = \eta \left( \Gamma \int_0^\infty x^2 f(x, b_0) x^{2j} e^{-x^2} \right),
\]

(IV.8)

where \( \eta = 2\alpha \omega_{\text{LO}}^3 (\omega_c/\omega_{\text{LO}})^{3/2} \), \( b_0 = b(\hbar/2m_b\omega_c)^{1/2} \) and the form factor \( f(k, b) = (8b^3 + 9b^2k + 3bk^2)/[8(b + k)^3] \).

In the case of zero LL broadening the magneto-optical absorption spectrum takes the form of a series of \( \delta \)-peaks at the positions determined by

\[
\omega - \omega_c - \text{Re} \sum(\omega) = 0,
\]

(IV.9)

and with the oscillator strength

\[
\pi \left[ 1 - \frac{\partial}{\partial \omega} \text{Re} \sum(\omega) \right]^{-1}.
\]

(IV.10)
Figure IV.1. The positions of the first four peaks in the magnetooptical absorption spectrum plotted as a function of the magnetic field for an ideal 2D system. Adapted from [64].

Numerically calculated frequencies in the [64] of the first four peaks in magnetoabsorption corresponding to polaron transitions from the ground state \((j = 0)\) to the LL \((j = 1, 2, 3, 4)\) when \(\Gamma = 0\) are shown in Fig. IV as a function of magnetic field. An anticrossing of \(\omega_1\) and \(\omega_2\) with \(\omega = \omega_{\text{LO}}\) is clearly seen, as well as a pinning of \(\omega_2\) to \(\omega_c\) at \(\omega \gg \omega_{\text{LO}}\).

V. COLLECTIVE OSCILLATIONS IN A TWO-DIMENSIONAL ELECTRON PLASMA

A. Plasmons

In solid state physics, plasmons are collective longitudinal oscillations of a charge density of a free electron gas in a restoring electric field created by an ionized lattice atoms. They can be excited by an external radiation impinging on the electron plasma and carrying enough energy and momentum. In 3D, plasmons are dispersionless while in 2D their energy changes as a square root of wavevector:

To derive equations for the oscillations of a 2D plasma we will use an electrodynamic model. We will consider a sheet of a 2DEG of zero thickness at the plane \(z = 0\), surrounded by a homogeneous 3D material with a static dielectric constant \(\varepsilon_{\text{ins}}\) for \(z < 0\) and \(\varepsilon_{\text{sc}}\) for \(z > 0\) [Fig. (V.1)]. For now, let us keep \(\varepsilon_{\text{sc}} = 1\). When an external electric field of the
form of a plane wave $E(k, \omega) = E \exp(i k \cdot r - i \omega t)$ is acting on the system, the density of electrons redistributes in space and induces an additional space charge $\rho_{\text{ind}}(r)$. Then, the electrostatic potential $\Phi(r)$ resulting from a spatial charge redistribution is found from the Poisson’s equation

$$\Delta \Phi(r) = -\rho_{\text{ind}}(r)/\varepsilon_0\varepsilon_{\text{ins}}.$$  \hfill (V.1)

The field induced by the potential $\Phi(r)$ screens the influence of the external field on a single electron in the 2DEG sheet:

$$E_{\parallel}(k, \omega) = E(k, \omega) - e\Phi(r).$$ \hfill (V.2)

Then the polarization of the 2DEG is $P(k, \omega) = \chi(k, \omega) E_{\parallel}(k, \omega)\delta(z)$, with a polarizability $\chi = \chi_1 + i\chi_2$ of an isotropic electron sheet with energy bands $\epsilon = (\hbar k)^2/2m_e^*$ and the Fermi wavevector $k_F$, where $\chi_1$ and $\chi_2$ \cite{65} are:

$$\chi_1 = G \left[ 2z - C_+ \sqrt{(z-u)^2-1} - C_- \sqrt{(z+u)^2-1} \right],$$ \hfill (V.3)

$$\chi_2 = \sigma/\omega = G \left[ D_+ \sqrt{1-(z-u)^2} + D_- \sqrt{1-(z+u)^2} \right],$$ \hfill (V.4)

$$G = n_e e/m_e^* z k_F^2 v_F,$$ \hfill (V.5)

$$C_+ = (z+u)/|z+u| \text{ and } D_+ = 0 \text{ for } |z \pm u| > 1,$$ \hfill (V.6)

$$C_- = 0 \text{ and } D_- = 1 \text{ for } |z \pm u| < 1,$$ \hfill (V.7)

$$z = k/2k_F, \ u = \omega/k_F.$$ \hfill (V.8)

Here, $\sigma$ is the conductivity term given by Eq. (III.6). The dielectric function for longitudinal excitations in the plane of the 2DEG, is
\[ \varepsilon(k, \omega) = \varepsilon_{\text{ins}} + 2\pi\beta\chi(k, \omega), \quad (V.9) \]

with \( \beta = \sqrt{k^2 - \varepsilon_{\text{ins}}\omega^2/e} \). Now, the frequency of the longitudinal modes in the system can be found from the condition \( \varepsilon(k, \omega) = 0 \). If \( \varepsilon_{\text{ins}} \) is not frequency dependent, \( u \gg 1 \) and for the long plasma wavelengths the relation \( k = \omega^2/a \) is valid, where \( a = n_e e^2/m^*\varepsilon_{\text{ins}} \). This results in a 2D plasmon dispersion relation \[65, 66\]:

\[ \omega_p = \sqrt{\frac{n_e e^2 k}{2\varepsilon_0 \varepsilon_{\text{ins}} m^*}}, \quad (V.10) \]

In most experimental cases, the 2D electron sheet is separated from the air by a barrier of thickness \( d \) which screens the electric field of incident electromagnetic wave. Also, a quite common situation is when a perfectly conductive plane (a metallic contact, for example) is placed on the top of the barrier for the control of the electron concentration in the channel. Figure (V.2) illustrates both situations. Plasmons excited in these two cases are referred to as ungated and gated plasmons, respectively. In order to incorporate screening effects in the 2D plasmon dispersion, the dielectric constant \( \varepsilon_{\text{ins}} \) in Eq. (V.10) is replaced with an effective dielectric function \( \bar{\varepsilon}(k, d) \). For the gated \( [\bar{\varepsilon}_g(k, d)] \) and ungated \( [\bar{\varepsilon}_\text{ug}(k, d)] \) plasmon cases, respectively, the following expressions of the effective dielectric functions are \[67, 68\]:

\[ \bar{\varepsilon}_g(k, d) = \frac{1}{2} \left[ \varepsilon_s + \varepsilon_b \coth(kjd) \right], \quad (V.11) \]

\[ \bar{\varepsilon}_\text{ug}(k, d) = \frac{1}{2} \left[ \varepsilon_s + \varepsilon_b \frac{1 + \varepsilon_b \tanh(kjd)}{\varepsilon_b + \tanh(kjd)} \right], \quad (V.12) \]

where \( \varepsilon_s \) is the static dielectric constant of the quantum well and \( \varepsilon_b \) is the static dielectric constant of the barrier. Comparing Eqs. (V.11) and (V.12) the effect of the gate electrode screening is:

\[
\left( \varepsilon_b(1 + \varepsilon_s) + \varepsilon_s \tanh(kjd) + \varepsilon_b^2 \tanh(kjd) \right) \left( \varepsilon_b(1 + \varepsilon_s) + \varepsilon_s \tanh(kjd) + \varepsilon_b^2 \coth(kjd) \right)^{1/2}.
\]

B. Magnetoplasmons

In the presence of an external magnetic field perpendicular to a 2D electron plane, plasmons couple to the electron cyclotron motion and form hybrid resonant excitations referred
Figure V.2. Gated and ungated structure with 2DEG layer situated under the barrier

to as the magnetoplasmons. In general, a description of magnetoplasmons should be carried
out within a non-local model of the magnetoconductivity tensor of an electron gas. The
derivation of a magnetoplasmon dispersion will follow the Refs. [69, 70].

An infinite sheet of a 2DEG is situated on a surface of a semi-infinite semiconductor and
is separated from the air by a barrier of a thickness $d$. A magnetic field induction vector is
normal to the plane of 2DEG. Basing on the Drude model, in a quasi-static approximation,
the magnetoplasma dispersion relation gets the form (see for example Ref. [71]):

$$\frac{2\varepsilon_0 \varepsilon(k, d) \omega_{mp}}{k} + i\sigma_{xx} = 0,$$  \hspace{1cm} (V.13)

with $\omega_{mp}$ - the resonant magnetoplasma frequency, $k$ - the magnetoplasmon wavevector
and $\sigma_{xx}$ - the component of longitudinal conductivity; $\varepsilon(k, d)$ is a dielectric function, which
depends on the thickness of the barrier and on the wavelength of a magnetoplasmon. According
to Refs. [69] and [70], an analytical expression for $\sigma_{xx}$ in the semiclassical limit
disregarding the electron-electron interaction is:

$$\sigma_{xx} = \frac{i n_s e^2 \omega_{mp}}{m_e^* X^2} \sum_{l=1}^{\infty} \frac{4l^2 J_l^2(X)}{\omega^2 - (l\omega_c)^2}. \hspace{1cm} (V.14)$$

Here $n_s$ represents a 2DEG concentration, $m_e^*$ stands for the electron effective mass, $X = k v_F / \omega_c = 2\pi r_c / \lambda_p$ is so called non-local parameter that quantifies the extent to which
cyclotron orbit $2\pi r_c$ matches the plasmon wavelength $\lambda_p$. $J_l(X)$ is a Bessel function of the
first kind and the order $l$. Magnetic field is incorporated in the cyclotron frequency term
$\omega_c = eB / m_e^*$ and in the cyclotron radius term $r_c = m_e^* v_F / eB$. The latter equation assumes
a collisionless limit; we use it because we are interested only in a dispersion which relates
resonant magnetoplasma frequencies with their wavevectors. Substituting Eq. (V.14) to Eq.
\( (V.13) \), we get a general dispersion relation for magnetoplasmons:

\[
1 - \frac{\omega_p^2}{X^2} \sum_{l=1}^{\infty} \frac{4l^2 J_l^2(X)}{\omega^2 - (l\omega_c)^2} = 0, \tag{V.15}
\]

with a quantity \( \omega_p \) standing for a 2D plasma frequency at zero magnetic field, which in a long wavelength limit \( (kd \ll 1) \) is given by Eq. \( (V.10) \).

In a local limit (the non-local parameter \( X \ll 1 \)), \( J_l(X) \) is asymptotically equal to \( (X/2)^l/\Gamma(l + 1) \) \[72\]. Taking this into account, the terms in the sum in Eq. \( (V.15) \) of higher order than \( l = 1 \) in the sum can be neglected. Putting \( J_1(X) \approx X/2 \) reduces Eq. \( (V.15) \) substantially and it becomes:

\[
1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} = 0. \tag{V.16}
\]

This is an expression of magnetoplasmon dispersion relation in the local limit for a classical two-dimensional electron gas. It was derived in Ref. \[73\] considering the magnetic field influence on a 2D Wigner crystal. A semiclassical derivation with the aid of many-body quantum mechanical calculations was demonstrated by Chiu and Quinn \[69\].
Part II

Samples and Experiment Setup

I. SAMPLES

An object of investigations in this thesis was a variety of CdTe/Cd$_{1-x}$Mg$_x$Te quantum wells (QW) containing a high mobility 2DEG which were grown on GaAs substrates and modulation doped at one barrier. Modulation doping was used to remove ionized impurity atoms from a 2D electron layer and to enhance the mobility of a 2DEG.

The samples were cut from wafers grown by a molecular beam epitaxy (MBE) technique on epiready 2" GaAs substrates of (100) orientation [74]. This particular orientation ensures crystallization of Cd$_{1-x}$Mg$_x$Te epilayers in a cubic zincblende structure, matching the structure of a CdTe crystal lattice. Ternary Cd$_{1-x}$Mg$_x$Te alloy was proved to be a suitable barrier material for a high electron mobility CdTe QW due to a small mismatch of lattice constants (within 1%) and a possibility to tune the bandgap up to 3 eV changing the Mg contents [75, 76]. As it is shown in Fig. (I.1) an epitaxial Cd$_{1-x}$Mg$_x$Te layer of thickness of a few microns was deposited on a GaAs substrate in order to reduce a dislocation density arising due to the lattice constant mismatch between the barrier and the substrate. A single CdTe quantum well is embedded between the barriers and is separated from an iodine doped Cd$_{1-x}$Mg$_x$Te layer by an undoped spacer. Iodine forms a hydrogen-like donors at the sites of tellurium. In general, other halogens such as Br$_2$ or Cl$_2$ can be used for $n$-type doping as well. However, it seems that incorporation of I$_2$ introduces the smallest distortions to the host lattice, being the most similar to tellurium in terms of the size and electronegativity [77]. Once again, lattice distortions in the barrier might introduce a strain in the quantum well and reduce the mobility of 2DEG. Additional doped layers are usually introduced in order to control the Fermi level position and to assure better electrical contacts to a 2D electron layer. Ohmic contacts were formed by soldering indium droplets at the temperature of $T = 180^\circ$C for approximately 30 seconds.

The main parameters of the wafers such as the quantum well width and magnesium content in the barrier are presented in Table I.

Samples with five types of the surface geometries were produced on the wafers. Two
Table I. Main parameters of CdTe/CdMgTe quantum wells.

<table>
<thead>
<tr>
<th>Wafer #</th>
<th>Well width [nm]</th>
<th>Mg cont. [%]</th>
<th>(n_{2D} \times 10^{11} ) [cm(^{-2})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>15</td>
<td>3.1</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>17</td>
<td>4.0</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>25</td>
<td>3.6</td>
</tr>
<tr>
<td>D(^*)</td>
<td>20</td>
<td>—</td>
<td>2.5</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
<td>30</td>
<td>4.2</td>
</tr>
<tr>
<td>F</td>
<td>20</td>
<td>33</td>
<td>3.1</td>
</tr>
<tr>
<td>G</td>
<td>15</td>
<td>33</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Figure I.1. (a): The configuration of the samples with two ohmic contacts for current transfer; (b) and (c): Wafer layer-by-layer composition of samples P1 and P2, respectively.

samples, P1 and P2, (wafers A and B, respectively) of the simplest form, are containing only two indium contacts to pass the current. Their surface schematics is shown in Fig. (I.1) (a) and the layer-by-layer wafer composition of the samples P1 and P2 is shown in Fig. (I.1) (b) and (c), respectively. The thickness of iodine doped layer is measured in monolayers (ML). In the case of CdTe, one ML corresponds to 3.24 Å. For a matter of simplification, later in the text these two contacts will be referred to as the drain and the source contacts.

Other two samples, P3 and P4, (wafer C) had a gate contact on the top, which enabled to control the electron concentration in the 2DEG channel. The gate was formed by depositing a stack consisting of 3 nm of chromium and 120 nm of gold. To decrease the gate leakage currents, the gate electrode was isolated from a surface of the wafer by a layer of a high-\(k\) material HfO\(_2\). A surface structure of the samples is shown in Fig. (I.2) (a) and the compo-
Figure I.2. (a): The top structure of the samples with two ohmic contacts for current transfer and gate contact; (b): Wafer layer-by-layer composition of samples P3, P4; (c): Sample P5 inner structure.

The sample P5 (wafer D) contains a CdMnTe quantum well. It was used to study the THz radiation detection in the gated 2DEG as a function of gate bias.

THz detection and photocurrent dependencies of the gate bias were investigated in a Hall bar sample (HB) formed on the wafer E by an electron beam lithography. A conduction channel and Hall contact paths were separated from the rest of the wafer by the etched trenches cutting the 2DEG sheet. The sample had a channel of $W = 50 \, \mu m$ width and $L = 100 \, \mu m$ length. The channel was covered with a gate contact stack composed of a 30 Å thick chromium and a 1200 Å thick gold layers. No dielectric layer was deposited between the gate metalization and the surface of the wafer. Drawing in Fig. I.3 (a) demonstrates the central part of the sample with the gate contact, conduction channel and Hall contacts paths. All conductive paths broaden when going from the central part towards the edges of the sample (not shown in the drawing) where indium contacts were formed. Figure (I.3) (b) shows the inner structure of the wafer.

In order to explore magnetoplasma excitations in a high electron mobility CdTe/CdMgTe quantum wells, a series of samples containing a metallic grid-gate on the top surface were processed [Figs. (I.4) (a) and (b)] on a wafer F [Fig. (I.4) (c)]. Photolithography and aluminum lift-off allowed to define a 1.6 mm × 1.6 mm grid coupler isolated from the surface of the wafer by a 70-nm-thick SiO$_2$ layer. A thickness of an aluminum grid was 110 nm. The coupler period $\Lambda$ was equal to 2 $\mu$m and 3 $\mu$m for the samples G1 and G2, respectively, and the geometrical aspect ratio of the grid was equal to 50% in each case. A
Figure I.3. (a): The gated Hall bar sample (HB) and (b): Wafer layer-by-layer composition.

Figure I.4. (a): Composition of the samples G1 and G2 containing the metallic grid on the top, (b): Grid fingers shown in detail, (c): The structure of the wafer.

part of the grid structure is shown as a drawing in Fig. (I.4) (b). A reference sample GR was prepared with neither a grid nor a SiO₂ layer.

The quantum point contact sample QPC [conduction channel is shown in a SEM photo presented in Fig. (I.5) (a)] was realized by an electron beam lithography on a wafer G [Fig. (I.5) (b)]. Etched trenches cut 2DEG layer and separate the channel (C) of a non-trivial form from two lateral gates (G). A nominal width of the channel $W$ is equal to 2.4 $\mu$m and decreases to 460 nm at the constricted part. The channel and gates are connected to large-surface areas, isolated from one another, were ohmic contacts were formed by soldering indium. These parts are not shown in the picture.
Figure I.5. (a): Scanning electron microscope image of the QPC sample showing the channel (C) and the split-gate structure (G). (b): The inner composition of the sample.

Figure II.1. The experimental setup for a DC magnetotransport measurements at low temperatures.

II. EXPERIMENT SETUP

The nature of physical phenomena investigated in this work requires low temperatures. To keep the samples at low temperatures an open cycle liquid helium cryostat from “Cryogenics” was used. It is equipped with a variable temperature inset (VTI), what enables to change the temperature of the sample from 1.5 K to roughly 200 K. An effective temperature stabilization is achieved by simultaneously adjusting helium gas flow through the VTI and an electrical current flowing through the heater coil. Temperature is measured by a calibrated “Cernox” resistor placed in a vicinity of the sample. In order to provide the magnetic field, the cryostat is equipped with a superconducting electromagnet coil immersed into the liquid helium bath and capable to induce a maximum magnetic field of 17 T. The bore of the coil is wide enough to accommodate a bottom VTI part inside.
The sample holder itself is a circular stainless steel waveguide, 11 mm in diameter, with a Winston cone attached at its end to focus the incoming THz radiation on the sample. The investigated sample is glued to a special holder and mounted just below an output slit of the Winston cone. When the sample holder is inserted into the cryostat, its top is exposed to room temperature, while the bottom reaches the center of the magnet coil. Wires running from the top to the bottom of the waveguide allow for electrical connections. The top of the waveguide is sealed with a teflon window. At the bottom of the waveguide, a black polyethylene “cold filter” is placed, which prevents a thermal radiation to reach the sample.

In this setup, experiments were performed in a Faraday configuration, i.e. with magnetic field lines and a wavevector of the incoming THz radiation normal to the surface of the sample.

A. Terahertz radiation sources

When measuring a spectrum in magnetic field, two approaches are commonly used: (a) radiation frequency is constant and the sweep of the magnetic field is performed; (b) the magnetic field is set constant and a scan through radiation frequencies is done. In our experiments, both approaches were applied. In the former case we used a THz or a far-infrared gas laser (FIRL) operating at constant frequencies; in the latter case – an infrared Fourier transform spectrometer (FTIR) was used.

1. A Terahertz laser

In the experiments, an optically pumped molecular THz laser “FIRL-100” produced by “Edinburgh Instruments” was used. The device is a system comprised of a gas FIR laser which is pumped by a CO\textsubscript{2} laser of a high output power. The CO\textsubscript{2} section is operated in a flowing gas mode in order to obtain output powers higher than 50 W on the strongest lines. The gain section of the laser is a single arm water cooled discharge tube sealed by ZnSe Brewster windows. The off-axis optical modes are effectively suppressed by a special design of the gain section and internal reflections are neglected by a profiled internal wall of the discharge tube. The resonator is realized by a partially reflecting ZnSe output coupling the mirror M\textsubscript{5} mounted on a piezo-transducer and a gold coated water cooled diffraction

31
Grating GR blazed for 10 µm. Actuation of piezo-transducer allows for a fine tuning of the output frequency. The laser is capable of giving over 80 stable lines in the 9-11 µm range.

The FIR section of the device is comprised of a ZnSe input window with a Brewster angle, which forms a vacuum seal at one end of the laser, a flat Cr-Au coated stainless steel input mirror M2 and a specially optimized dichroic output coupler M1 from which a THz radiation is extracted. The length of the cavity can be changed by moving the output mirror mounted on the moving stage operated by a servo motor. The gain medium is kept in a oversized Pyrex waveguide of 36 mm inside diameter. The pump energy is coupled through a 2 mm hole in the center of the input mirror (M2). The FIR laser can operate in a 40-1222 micron region, which corresponds to frequencies in a 0.25-7.5 THz interval. A nominal power of 118.8 µm (2.52 THz) methanol line is 120 mW. The output radiation is polarized either parallel or perpendicular (depending on the gain medium gas) to the input pump laser polarization.

In the experiments, the laser was operated in a cw mode, with the methanol gas as the active medium. This allowed us to have four stable lines at 96.5 µm (3.11 THz), 118.8 µm (2.52 THz), 163 µm (1.84 THz) and 186 µm (1.61 THz) with a power from 1 to 30 mW. A THz radiation at the laser output was mechanically modulated and fed to the horn of an oversized circular metal waveguide. Afterwards, it was directed onto the sample placed in a liquid helium cryostat as it was described in Sec. II. A laser power was continuously measured during the experiments with a pyroelectric detector. A signal from it was acquired by a lock-in amplifier at the sampling frequency of the laser radiation modulation.

When a photocurrent was investigated, the sample was connected in series to a load resistor. This electrical circuit was biased with a dc voltage, and an ac voltage drop in the resistor was measured and digitalized by a conventional lock-in technique at the frequency of the mechanical chopper. A digital data from the lock-in amplifier buffer was sent to a personal computer (PC) through an IEEE-488 (GPIB) communication bus and visualized with a help of a data acquisition program written in the “LabView” programming language.

A signal of transmission spectra was gathered by a thinned carbon resistor serving as a bolometer. The load resistor and a bolometer were biased with a voltage source and a voltage signal was picked at the modulation frequency from the load resistor. Spectra were visualized in a PC screen as in the case of photocurrent measurements.
Figure II.2. Experiment scheme for low temperature THz magnetospectroscopy in transmission and/or photocurrent mode. Optically pumped FIR laser is used as a THz source.

2. An infrared Fourier spectrometer

An infrared Fourier spectrometer “Bruker IFS 66V/s” located in Charles Coulomb Laboratory at Montpellier University 2, Montpellier, France, was used to carry out measurements as a function of radiation frequency. An essential element of the spectrometer is a high throughput Michelson interferometer, which creates interferograms of the optical signal at the input by a moving mirror at one of the arms. A radiation is sent to the sample where it is modulated and the resulting signal is detected in absorption or in reflection. Finally, with the help of an internal electronics, the detector output is digitalized and Fourier transformed to a spectrum in a frequency domain. The Spectrometer was evacuated in order to avoid spectral lines coming from the absorption of atmospheric water vapour. “Bruker IFS 66V/s” spectrometer covers a broad frequency range of 20-14000 cm$^{-1}$ (0.6-420 THz) with a maximum resolution of 0.25 cm$^{-1}$. However, our needs were limited to frequencies in the
Figure II.3. Experiment setup for a low temperature THz magnetotransmission with infrared Fourier transform spectrometer.

interval of 20-300 cm$^{-1}$ and a resolution between 2 cm$^{-1}$ and 0.5 cm$^{-1}$.

A configuration with a Globar source, a 6-µm-thick Mylar beamsplitter and a helium cooled Ge bolometer was used in the experiments. The bolometer was located in a cryostat described in Sec. II in a special chamber isolated from the helium bath at the bottom of a VTI and sealed with a quartz window. A heat transfer took place through a cooper heat link on which the bolometer was mounted. Thus, the temperature of the bolometer was about 7-8 K. Also, the bolometer was magnetic field-compensated. The radiation from the spectrometer output was guided on the spherical mirror (10 cm in diameter) and then sent to the bolometer by an oversized waveguide. On the way to the bolometer, the information carried by the
interferogram was modified by transmission through the sample. Bolometer output was connected to spectrometer’s inner computer, where the spectrum in the frequency domain was prepared. The spectrometer itself was controlled by a PC where various spectrometer parameters were set and the final spectra were shown. In this manner, transmission spectra in the magnetic fields from 6 T to 16 T were registered. Every measured spectrum was averaged on 99 samples in order to reach a better signal-to-noise ratio.
Part III

Results and Discussion

I. MAGNETOTRANSPORT

Magnetotransport measurements at low temperatures provide an information essential for sample characterization. For example, an electron mobility could be evaluated from the low field magnetoresistance. The electron concentration in a 2DEG is determined from the resistivity (conductivity) oscillations (a Shubnikov-de Haas effect) periodic in the inverse magnetic field, while the effective mass and Landau level broadening might be deduced analyzing an amplitude of the oscillations.

Usually, magnetotransport data is measured in four-probe configuration which helps to avoid an influence of a contact resistance. However, in this thesis magnetotransport was measured in a two contact configuration, and the concentration of a 2DEG was the main parameter of interest. Another limiting reason was difficulties in preparing good quality contacts on CdTe/CdMgTe heterostructures with indium soldering. Also, the investigated samples with the length of the channel larger than the width ($L > W$), were not suited for magnetotransport measurements. Our experiments were performed in temperatures from 1.8 to 8 K and in quantizing magnetic fields up to 16 T.

A. A Shubnikov-de Haas effect

1. An electron concentration and an effective mass

A typical example of a resistance dependence on magnetic field is plotted in Fig. (I.1) (a). During the experiments, a drain-source current $I_{ds}$ was registered while applying a constant voltage $V_{ds} = 20$ mV to the drain contact and keeping the source on the ground. At low magnetic fields, a classical DC magnetoresistance is observed. Increasing $B$, the oscillations of magnetoresistance arising from quantum effects are observed along with a DC magnetoresistance. Oscillations were proven to be periodic in the inverse magnetic field, therefore arising from the Shubnikov-de Haas effect. In order to determine the frequency of the Shubnikov-de Haas oscillations, a Fast Fourier Transform (FFT) procedure was applied.
Figure I.1. (a) A typical magnetoresistance curve of the sample P2 at 1.8 K. (b) The oscillating part of magnetoresistance in inverse $B$. It was symmetrized around zero field sample resistance by subtracting a numerically evaluated $R_{xy}^{dc}(B)$ from the curve shown in (a).

to the oscillating part of the magnetoresistance in inverse $B$ [Fig. I.1 (b)] for samples P1 and P2. In order to avoid a low frequency component manifestation in the FFT spectrum, a baseline which represents a DC part of magnetoresistance was defined numerically and subtracted from the measured spectra. To get the baseline, the measured magnetoresistance curve was smoothed numerically using the Tikhonov regularization algorithm. A sheet electron concentration $n_{s1} = 3.1 \cdot 10^{11}$ cm$^{-2}$ and $n_{s2} = 4.0 \cdot 10^{11}$ cm$^{-2}$ for samples P1 and P2, respectively, was found with the aid of the Eq. (III.12). Power spectra of FFT demonstrating
two narrow peaks are shown in Fig. (I.2). Inverse magnetic field period $\Delta(1/B)$ is given by the position of the lower (main) frequency peak. The peak at higher frequency represents the second harmonic of the main frequency and originates from a splitting of LLs into two levels with the opposite electron spin polarization, also known as the Zeeman effect.

The electron effective mass $m_e^*$ and the quantum scattering time $\tau_q$ can be deduced by analyzing the magnitude of SdH oscillations. To describe the absolute oscillation amplitude at relatively low magnetic fields, Ando [78] derived a formula for an amplitude envelope function, which reads:

$$\frac{\Delta R}{4R_0} = \frac{A}{\sinh(A)} \exp(-\pi/\omega_c \tau_q).$$  \hfill (I.1)

Here $A = 2\pi^2 kT/\hbar \omega_c$, $\omega_c$ stands for the cyclotron frequency, $R_0$ is the sample resistance at zero magnetic field and $\Delta R = R_a - R_0$ where $R_a$ is the amplitude of a SdH peak (dip).

The quantity $\tau_q$ is called a quantum (total) scattering time and is inversely proportional to scattering events occurring at all angles. On the contrary, the transport scattering time $\tau_c$ which is weighted by the factor $1 - \cos(\theta)$, where $\theta$ is the scattering angle, takes into account only scattering at small angles (scattering from distant impurities, for example). Therefore, the ratio $\tau_c/\tau_q$ gives an information about a relative importance of a small-angle scattering versus large-angle scattering. The time $\tau_c$ can be extracted from the Hall effect measurements at low $B$.

Magnetotransport experiments were done at temperatures in a 2–8 K range and results at
a weak field are presented in Fig. (I.3). Here, the oscillating part of magnetoresistance was symmetrized around zero by subtracting the classical (non-oscillating) $R_{xx}$. A magnetic field dependence of oscillating $R_{xx}$ was estimated numerically with a help of before mentioned Tikhonov regularization. The envelope function of SdH oscillation amplitude at low magnetic fields where spin-polarized LLs are not resolved yet, was described using the Ando formula [Eq. (I.1)]. In order to estimate time $\tau_q$, it was modified in the following way:

$$\log \left( \frac{\Delta R \sinh(A)}{4R_0 A} \right) = -\frac{\pi}{\omega_c \tau_q}. \quad (I.2)$$

Plotting the left hand-side of the latter expression as a function of inverse $B$ [Fig. (I.3) (a)] the so called Dingle plot [79] was obtained for samples P1 and P2 at $T = 2$ K. The resulting curves were then fitted with the Eq. (I.2) [solid lines in Fig. (I.3) (a)] and from the slope the quantum scattering time $\tau_q$ was found to be 0.57 ps and 0.52 ps for samples P1 and P2 respectively, taking $m^*_e = 0.1m_0$. These values agree well with the results for a high mobility 20-nm-wide CdTe quantum wells found in the literature [32, 80]. The same references also report on a significant increase of $\tau_q$ to $\sim 3$ ps after illuminating the sample with a visible light.

Also, theoretically expected density of states quantization is found in real situations, when LLs are broadened. The time $\tau_q$ is related to a LLs broadening. Assuming that a broadened LL shape is estimated by a Lorentzian and $\tau_q$ does not depend on the energy or magnetic field, the broadening $\Gamma = \hbar/2\tau_q$ can be estimated. In our case it turned out that for both specimens $\Gamma \approx 600 \mu$eV in a range of magnetic fields where $\tau_q$ was estimated.

At stronger fields, a deviation from a theoretically suggested linear dependence was observed in a Dingle plot at condition $\omega_c \tau_q \gtrsim 1$. This observation coincides with result of Coleridge et al. [79] suggesting a linear behaviour of $\Delta R_{xx}/R_0$ with a reciprocal magnetic field only until $\omega_c \tau_q \sim 1$ condition is reached.

Due to the geometry of experiments, it was not possible to define a reliable value of the transport scattering time $\tau_c$ using the expression for a classical geometrical magnetoresistance $R_{xx} = R_0(1 + \mu^2 B^2)$.

The ratio $\Delta R/R_0$ calculated for the sample P1 at different temperatures at the resistance minima at Landau level filling factors $\nu_1 = 16$ ($B_1 = 0.93$ T) and $\nu_2 = 14$ ($B_2 = 1.04$ T) is shown in Fig. (I.3) (b) as circles and triangles, respectively. Since $R_0$ varies slightly with $B$, the values of $R_0$ at the SdH minima corresponding to the filling factors mentioned
Figure I.3. (a): Dingle plots for samples P1 and P2 (circles and triangles, respectively) at $T = 2$ K. Solid lines are linear fits of Eq. (I.2) to the data. Quantum scattering time was deduced from the slope of linear fits. (b): Ratio $\Delta R/R_0$ for the sample P1 as the function of temperature at Landau level filling factors $\nu_1 = 16$ ($B_1 = 0.93$ T) and $\nu_2 = 14$ ($B_2 = 1.04$ T) marked as open dots and triangles, respectively. Solid curves represent least square fits with Eq. (I.1).

before, were determined by averaging $R_a$ at the SdH minimum and a mean of $R_a$ calculated at two adjacent maxima. Fitting numerically [solid curves in Fig. (I.3) (b)] the obtained temperature dependencies of $\Delta R/R_0$ with the Ando formula [Eq. (I.1)], the effective mass in the sample P1 (20 nm CdTe quantum well) was estimated. At a filling factors $\nu_1 = 16$ and $\nu_2 = 14$ the effective mass was found to be $m_{e1}^* \approx 0.103m_0$ and $m_{e2}^* \approx 0.095m_0$, respectively.
The latter results are consistent with a value of cyclotron effective mass $m^*_e \approx 0.102 m_0$, determined from the THz magnetospectroscopy on the same sample, when an increase in a mass due to a resonant polaron effect is neglected (see subsection (III B)).

Now, let us come back to the spin resolved Landau levels. A crude evaluation of the electron Landé factor $|g|$ is possible by comparing energies of a Zeeman gap in the field $B_1 = 2.6$ T (SdH minima is resolved for the odd $\nu$), and of a cyclotron gap at $B_2 = 0.5$ T (the first SdH minimum observed for even $\nu$), i.e. by solving the equation $g = e\hbar B_1 \mu_B m^*_e$, where $\mu_B$ is Bohr magneton. The enhanced $g^* \approx 3.8$ value was found (taking $m^*_e = 0.102 m_0$), significantly higher than a bare one for electrons in CdTe ($|g| = 1.6$) [81, 82]. However, our value is in a good agreement with the one obtained by Kunc et al. [80] in a 20 nm width CdTe QW with a high electron mobility, where the enhancement of $g^*$ at odd filling factors was measured by polarization-resolved magnetophotoluminescence. Authors argue that a spin gap increase is governed by a Coulomb interaction in a 2DEG. In a theoretical treatment of the problem, an increased spin gap at odd LLs is seen as a manifestation of many-body effects arising from an electron-electron interaction in a 2DEG [78].

2. The Shubnikov-de Haas effect in the gated structures

The gate contact on the top of the heterostructure allows for a precise control of the electron concentration in the 2DEG channel. The period of SdH oscillation should change with the gate voltage. However, in cases when the gate electrode covers only a small part of the channel, it influences only the concentration of electrons localized under the gate. As we have investigated samples with big surfaces covered by a gate only partially, and the gated parts are much smaller than access (ungated) parts, this is an important issue which must be addressed. In Fig. (I.4) (a) the magnetotransport curves measured on a gated sample with an areal ratio of gated to access parts roughly equal to 1/4, are shown at different gate voltages. As it can be deduced after a Fast Fourier transform (FFT) procedure, no significant change in the electron concentration with the gate voltage is observed [Fig. (I.4) (b)]. Only an increase in the channel resistance with changing $V_g$ towards the threshold voltage ($V_{th} = -0.8$ V) is seen. It seems that in such cases, a conventional DC magnetotransport measurement technique, when a constant drain-source voltage $V_{ds}$ is applied to the channel and a resulting current $I_{ds}$ is registered, is not sensitive to the concentration change under
Figure I.4. (a): Magnetotransport spectra measured at $V_g$ from 0 V (bottom curve) to -0.7 V (top curve) at every -0.1 V. (b): The FFT done on the magnetotransport spectra in an inverse magnetic field for the gate voltage in the range from 0 V to -0.6 V. The dip at the lower frequency in FFT spectra represents a period of a Shubnikov-de Haas oscillations arising from non-spin splitted Landau levels and defines the electron concentration $n_s = 4.51 \times 10^{11} \text{cm}^{-2}$ of 2DEG. The dip at the higher frequency is the second harmonic of the first one and represents spin resolved Landau levels. Positions of the peaks do not change with $V_g$. Spectra were shifted on $y$–axis for convenience.

the gate electrode. To detect the gate voltage influence on “gated” electron plasma, an improved DC measurement technique is used, when together with a constant $V_{dc}^g$, a small-amplitude voltage modulation in time $V_{ac}^g \sin(\omega_g t)$ is applied to the gate contact, while $V_{ds}$ is kept constant. In fact, the measured quantity is not the current $I_{ds}$ anymore, but rather
its derivative with respect to the amplitude of AC gate voltage part \( dI_{ds}/dV_{ac} \). To measure the derivative, a phase-sensitive detection at a frequency \( \omega_g \) is required. This experimental method is very useful when a local change in the concentration under the gate is a subject of concern, as it is illustrated in Fig. (I.5). In a plot (a), the spectra of \( dI_{ds}/dV_{ac} \) in magnetic field are shown for a constant gate voltage \( V_g \) from 0 mV to -600 mV at a step of -100 mV and with a sine modulation with a peak-to-peak amplitude \( V_{ac} = 20 \text{ mV} \) and a frequency \( f_g = 1533 \text{ Hz} \). The drain-source voltage \( V_{ds} = 40 \text{ mV} \). Changes in Shubnikov-de Haas oscillations pattern are clearly seen [compare with Fig. (I.4) (a)]. Figure (I.5) (b) illustrates how the electron concentration changes under gate electrode. At \( V_{dc}^{g} = 0 \text{ mV} \), \( n_s \) is slightly less than that defined from DC measurements and is equal to \( 4.16 \times 10^{11} \text{ cm}^{-2} \), and reduces to \( n_s = 1.73 \times 10^{11} \text{ cm}^{-2} \) at \( V_{dc}^{g} = -700 \text{ mV} \). Surprisingly, a gate leakage current \( I_{gs} \) was negligibly small, despite of the fact, that no insulator layer was added on the channel surface before depositing the gate metalization.

B. A fractional quantum Hall effect

In Fig. (I.6) (a) quantum oscillations in \( R \) (with a baseline stemming from Hall resistance digitally removed) are shown for the sample P5 containing a CdMnTe/CdMgTe heterostructure with Mn contents of 0.3% at temperatures \( T = 1.8 \text{ and } 4.2 \text{ K} \). A sheet carrier density obtained from Shubnikov-de Haas oscillations in this sample is \( n_s = 2.6 \times 10^{11} \text{ cm}^{-2} \). It is seen clearly, that at the temperature of 1.8 K a minimum in the resistance develops at the filling factor \( \nu = 3/2 \). We attribute this phenomenon to a fractional quantum Hall effect (FQHE) [50] developed at the upper spin branch of the lowest LL \( (N = 0) \).

For a high mobility GaAs/AlGaAs heterostructures at a temperatures of a few hundred mK, a variety of quantum Hall fractions were observed around filling factor \( \nu = 3/2 \), with a denominator of the fraction as high as 11 and was explained by a model of a spin-polarized composite fermions (CF) [55]. Also, the FQHE states at \( \nu = 5/3, 8/5, 7/5, 4/3 \) in Cd(Mn)Te/CdMgTe QWs were reported recently [33]. Authors of the work also show the possibility to tune the fractional gaps around \( \nu = 3/2 \) with a help of magnetic field through a \( s - d \) exchange interaction. The absence of the higher fractions when compared to the results on GaAs-based heterostructures, is most probably due to a lower quality of the material.

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Figure I.5. (a): Magnetotransport spectra measured on the gated Hall bar sample under the gate electrode for a DC gate voltages $V_{dc}^{g}$ between $-600$ mV (top curve) and 0 mV (bottom curve) every 100 mV and with a gate voltage modulation on. (b) FFT spectra at $V_{dc}^{g} = -700$...0 mV at every 100 mV. The peak at a SdH frequency from which $n_{s}$ is defined moves with a gate voltage. The sheet concentrations corresponding to $V_{dc}^{g}$ at 0 mV and at $-700$ mV are indicated.

We have also observed a dip in the resistance at $\nu = 3/2$ in the experiments on samples cut from CdTe quantum wells, showing that development of this state should not be attributed to magnetic ions (Mn). Piot et al. [32] reported on the fractions of 4/3 and 5/3 in the case of CdTe/CdMgTe QW.

The magnetotransport as a function of filling factor is shown in Fig. (I.6) (b) for the sample P2 with $n_{s} = 4.0 \times 10^{11}$ cm$^{-2}$ and a cyclotron mobility similar to that of sample P5.
Figure I.6. The magnetoresistance of samples P5 (a) and P2 (b) as a function of filling factor for the temperatures of 1.8 and 4.2 K. A FQH state of 3/2 is marked by the arrow and inscription.

Here, a distinct FQH state is resolved at $T = 1.8$ K and is not observed at $T = 4.2$ K; the behaviour is identical to the case of the sample P5 with incorporated magnetic ions. Also, well-developed dips at odd filling factors are observed for the lower temperature, resulting from a better defined gap between spin-polarized Landau levels.

Unfortunately, due to a lack of Hall contacts we were not able to measure Hall resistance. Also, the absence of ability to use milliKelvin temperatures kept us from detailed investigation of the FQHE in our samples.
Figure II.1. Transmission spectra in wavenumber $\tilde{\nu}$ domain measured on a reference sample GR by an infrared Fourier spectroscopy as a function of the magnetic field from 6 T to 15 T at every 1 T at the temperature of 4.2 K. Triangles mark the positions of cyclotron resonance for different filed induction. Spectra are shifted in the $y$-axis direction for clarity.

II. POLARON EFFECT

Magnetotransmission experiments in the THz domain were carried out at liquid helium temperatures with an infrared Fourier spectrometer and a terahertz laser. We have measured transmission through samples prepared on the wafer F, featuring a 2DEG in a 20-nm-wide CdTe quantum well. Fundamental physical phenomena, such as the cyclotron resonance, an electron effective mass increase due to polaron effect and oscillations of a high mobility magnetoplasma were observed and successfully explained in the frame of existing theories.

Transmission spectra obtained with an infrared Fourier spectrometer were collected in quantizing magnetic fields as a function of radiation frequency at $T = 4.2$ K. The spectrometer components such as the radiation source (a hot tungsten wire, MIR) and a beamsplitter (Mylar 6 $\mu$m) were carefully chosen to the optimal performance in the spectral interval of far-infrared wavelengths in which we were interested.

Each spectrum was recorded at a constant magnetic field, with the radiation frequency resolution of 1 cm$^{-1}$. The sample was kept at a small angle to the incident radiation in order to avoid interference effects in the substrate. However, fringing pattern coming from Fabry-Pérot interferences occurring in the quartz window which seals the bolometer compartment from the VTI in the cryostat, was present in final spectra. To make the spectra smoother,
the fringing frequency was determined and filtered-out numerically by carefully processing carefully interferogram data. Then, transmission spectra in magnetic field \((T_B)\) were divided by a transmission spectrum taken at \(B = 0\ T (T_0)\), in order to distinguish between a common background and the effects induced by an external magnetic field. This approach also helped us to exclude an influence of a spectral power distribution of spectrometer radiation and a spectral characteristic of the beamsplitter.

The resulting spectra (shifted in the direction of \(y\)-axis for clarity) are presented in Fig. (II.1) as a function of radiation energy in the range from 50 cm\(^{-1}\) to 140 cm\(^{-1}\) (1.5 THz – 4.2 THz) and in magnetic fields in a 6–15 T range at every 1 T. A cyclotron resonance dips, marked by triangles in a plot, are clearly seen in the spectra. The cyclotron effective mass was deduced to be \(m_c^* = (0.101 \pm 0.0005) m_0\) for a 20 nm-wide CdTe/CdMgTe quantum well, from the zero-intersecting slope of the linear part of \(\tilde{\nu}(B)\) function [Fig. (II.2)]. It is in agreement with the effective mass values obtained from magnetotransport (see subsection IA 1) as well as with data reported in the literature [39, 80]. What is more, at high magnetic fields, a reduction in the CR amplitude was also observed. We believe that both effects result from a resonant electron-optical phonon interaction (resonant polaron effect). Indeed, Ref. [64] showed that when \(\omega_c\) approaches longitudinal optical phonon frequency \(\omega_{LO}\), the CR oscillator strength is reduced.

Also, spectral features with the amplitude comparable to that of CR peaks appear in the region around 60 cm\(^{-1}\). However, they do not show any clear pattern, nor they were repeatable at different experimental runs. The observations suggest that these features do not carry any essential physical information, but in order to confirm it more experiments are needed.

A. Resonant polaron effect

The electron effective mass defined in previous sections by means of magnetotransport and THz magnetospectroscopy is not a real value of the bare electron effective mass \((m_b^* = 0.092\) for CdTe) due the to polaron effect. Polaron effect is pronounced strongly in the case of ionic crystals like CdTe, for instance, as an electron-lattice ion Coulomb interaction is stronger than in covalent crystals like Si or Ge.

In two dimensions, the polaron mass is defined as \(m_p^* = m_b(1+\pi\alpha/8)[83]\) when \(\omega_c \ll \omega_{LO}\).
Here $\alpha$ is a Fröhlich constant (in the case of CdTe, $\alpha = 0.3$) and $\omega_{LO}$ is the LO phonon frequency ($170.2 \text{ cm}^{-1} = 2\pi \cdot 5.1 \text{ THz}$ for CdTe). If $\omega_c \gg \omega_{LO}$, the electron mass decreases (but does not reach the value of $m_b$) because an electron in cyclotron motion moves so fast that the much slower phonon cloud is not capable to follow it. However, in the case of 2DEG polaron effect is smaller than predicted by $m_p^*$ and $m_b$ relation given above, due to electron screening effects.

A resonant polaron effect is observed, when $\omega_c \approx \omega_{LO}$. The $\omega_c$ is in anticrossing with $\omega_{LO}$ and splits into two branches with energies $\hbar \omega_{p1}$ and $\hbar \omega_{p2}$, respectively. The $\hbar \omega_{p1}$ is below and $\hbar \omega_{p2}$ is above the LO-phonon energy. The cyclotron effective mass $m_c^*$ in the branch above LO-phonon energy is lower than $m_p^*$, because the electron gains more velocity and lattice deformations fail to follow its motion. Nevertheless, $m_c^*$ is still higher than the bare electron mass at the bottom of the band, due to a band non-parabolicity.

Experimentally, the condition for the resonant polaron effect is usually reached by increasing the magnetic field. Performing scans with the excitation frequency in strong enough magnetic fields, it is possible to observe a splitting of cyclotron energy into upper and lower polaron branches. The phenomenon can be observed in a 2DEG hosted in polar semiconductors where the energy gap $\Delta = \hbar (\omega_{p2} - \omega_{p1})$ is larger than the reststrahlen band, also with a low electron bare effective mass, a relatively high Fröhlich constant and a small LO-phonon energy, for example, in InSe [84]. Similar experiments were successfully performed on CdTe/CdMgTe quantum wells but with spin resolved polaron energies [39].

Figure (II.3) shows a cyclotron energy dependence on the magnetic field for samples GR and G1 (both cut from the same wafer) taking cyclotron resonance positions marked by triangles in the transmission spectra in Fig. (II.1). In the case of the GR sample, the spectra were recorded every 0.25 T (open squares), while for the sample G1 – every 1 T (full circles). Nevertheless, it is clear from the graph that both dependencies follow the same path, thus G1 was not investigated with the higher $B$ resolution. The linear part of the experimental $\omega_c(B)$ dependence was fitted with $\omega_c = eB/m_c^*$, and $m_c^*$ was estimated to be $(0.101 \pm 0.005)m_0$. The approximated theoretical curve of $\omega_c$ is plotted as a dashed line. A deviation from the linear behaviour is clearly seen above $\sim 9.5 \text{ T}$. As the cyclotron energy approaches $\hbar \omega_{LO}$, we believe that we observe the manifestation of the resonant polaron effect. Unfortunately, the maximum magnetic induction available in our experimental setup was not enough to reach the condition for the resonant polaron effect, thus we have observed
Table II. Parameters for InSe and CdTe needed to compare the polaron effect in these materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \alpha )</th>
<th>( \hbar \omega_{\text{LO}} ) [meV]</th>
<th>( m^*_e/m_0 )</th>
<th>( B_{\text{LO}} ) [T]</th>
</tr>
</thead>
<tbody>
<tr>
<td>InSe</td>
<td>0.29</td>
<td>27.2 [86]</td>
<td>0.13 [87]</td>
<td>33.21</td>
</tr>
<tr>
<td>CdTe</td>
<td>0.30</td>
<td>20.8 [88]</td>
<td>0.101</td>
<td>18.85</td>
</tr>
</tbody>
</table>

only the lower polaron branch.

A common approach to describe magnetopolaron behaviour theoretically is to calculate the so-called memory function [63, 64, 85]. However, to solve Eq. (IV.9) is not a straightforward task. Therefore, to describe our experimental results in Fig. (II.3) we will compare them with memory function calculations found in the literature. As the reference data we will use calculations of Nicolas et al. [84], shown in Fig. (II.3) (a). The authors of Ref. [84] found an experimental \( \omega_c \) spectrum of InSe in magnetic field showing the cyclotron resonance energy splitting, as well as anti-crossing with \( \omega_{\text{LO}} \) energy and pinning of \( \omega_{p1} \) and \( \omega_{p2} \) to \( \omega_{\text{LO}} \) at the condition \( \omega_c \approx \omega_{\text{LO}} \). Also, they have calculated theoretical spectra for 3D, 2D and quasi-2D electron cases. As we have measured only the lower frequency branch of the polaron energy \( \omega_{p1} \), we were interested in the \( \omega_c < \omega_{\text{LO}} \) energy range. In order to compare our results with these calculated for InSe, the curve corresponding to calculations for a quasi-2D electron case was replotted in normalized units. The resulting curve is shown in Fig. (II.3) (b) with the units changed in the following way:

\[
\omega(B) \rightarrow \frac{\omega_c(B)}{\omega_{\text{LO}}}, \quad B \rightarrow \frac{B}{B_{\text{LO}}}. 
\]

\( B_{\text{LO}} \) is the magnetic field at which the LO frequency is equal to the cyclotron frequency if \( \omega_c(B) \) is expected to be linear, that is \( \omega_{\text{LO}} = eB/m^* \). The parameters for InSe and CdTe are presented in Table II.A. It is important to note that InSe has a wurzite crystal lattice, therefore the values of parameters in the direction perpendicular to \( c \)-axis were taken.

Next, the curve was “shifted” to higher energies, multiplying it by the factor \( \alpha_{\text{CdTe}}/\alpha_{\text{InSe}} \) in order to account for the Fröhlich constants discrepancy for InSe and CdTe. This kind of manipulation is allowed by a simple observation that according to Eqs. (IV.6) and (IV.8), \( \alpha \) is a constant multiplication factor in the term \( \text{Re} \sum \omega \) in Eq. (IV.9). Squares in Fig. (II.3) (b) mark our experimental results which were put in normalized units. From the plot, a good agreement of experimental results and theoretical results (solid curve) calculated for
Figure II.2. CR energy as a function of $B$ at every 0.25 T in the interval of 6–14 T. Here, the open squares represent experimental results for sample GR. Full dots show the results on sample G1. Dashed curve is the linear fit representing cyclotron resonance frequency $\omega_c = eB/m^*_c$ with fitting parameter $m^*_c$, which was found to be 0.101$m_0$. Solid curve shows cyclotron frequency when corrections due to polaron effect are taken into account (from [84], see the text).
Figure II.3. (a): Energy of upper and lower polaron branches for 2DEG in magnetic field in InSe [84]. Experiments were performed at $T = 10$ K. Dashed line – theoretical calculations for a 3D case, solid – quasi 2D case, dotted – 2D case. (b): Red solid curve shows the energy of lower polaron branch in quasi-2D case taken from picture (a) in normalized units (see the text). Blue squares represent the experimental points from Fig. II.2 in normalized units.

A quasi-2D InSe case is clearly visible. As it was stated in the original work, the $\omega_c$ curve for InSe was calculated taking $d = 36$ nm as a 2D layer effective thickness. Unfortunately, we do not know the effective thickness of 2D electron sheet in our samples.

In Fig. (II.4), the cyclotron energy dependence on the field from Fig. (II.2) was redrawn as an effective mass function of $B$. The cyclotron effective mass starts to increase quite
Figure II.4. Cyclotron mass as a function of magnetic field for the sample GR. An increase in $m^*_c$ at $B \sim 6.5$ T occurs at a Landau level filling factor $\nu = 2$.

significantly at $B \approx 10$ T; this is an important conclusion, which has to be taken into account when cyclotron resonance experiments are performed in the fields stronger than this value.

At $B \approx 6.5$ T a bump in an amplitude obviously higher than the margin of error in $m^*_c(B)$ is observed. The following behaviour can be understood as a “polaron effect” of Landau levels [37, 38, 89, 90], that is an effective electron mass increases at an integer LLs filling factors due to the differences in polaron effect at every LL. Assuming that our observed mass increase occurs at $\nu = 2$, the concentration of 2DEG in the sample GR can be extracted. The calculations give the value $n_s = 3.15 \times 10^{11}$ cm$^{-2}$, which is in agreement to the one provided by Hall measurements in the dark at $T = 1.8$ K ($n_s = 3 \times 10^{11}$ cm$^{-2}$). In our experiment the sample was additionally unintentionally irradiated with red light coming from a He-Ne laser used for spectrometer calibration. It will be shown later on that this enlightenment with a visible light turns out to be crucial for transmission experiments on samples with a grid coupler.
In this chapter, a terahertz radiation induced photocurrent registered on various samples based on CdTe/CdMgTe will be discussed. In the current experiments, the lines at a few different frequencies from the molecular THz laser operating in a cw mode were used to excite the photocurrent at temperatures of 1.8 K and 4.2 K. A resistor kept at room temperature was connected in series to the sample and a DC voltage biased the circuit. A mechanically modulated THz radiation was incident on the sample, and the photocurrent signal was recorded with a phase sensitive detector at a chopping frequency as an ac voltage drop on the load resistor.

A. A cyclotron resonance in the photocurrent

We have investigated a THz photocurrent in two samples P3 and P4 with a 30-nm-wide CdTe quantum well and in the configuration with two current contacts. Experiments were performed in magnetic fields at temperature $T = 4.2$ K and with a monochromatic laser radiation of wavelengths of 186 $\mu$m, 163 $\mu$m and 118.8 $\mu$m. Samples were biased with a dc voltage $V_{ds} = 150$ mV. By keeping $V_{ds}$ constant and guiding the chopped THz laser beam to the sample, we measured a difference between the current which flows in the sample when it is irradiated by the THz light, $I_r$, and the current in the dark, $I_d$. The results are presented in Fig. (III.1) (a) and (b) for the samples P3 and P4, respectively.

Sharp cyclotron resonance peaks at almost flat zero background in the photocurrent were observed for each laser line used. The background of spectra for $\lambda_0 = 163$ $\mu$m is influenced by a nonresonant photoresistivity. The insets in the figures show the position of a CR peak in magnetic field, which were fitted linearly with a dependence $\omega_c = eB/m^*_c$, and the electron cyclotron mass $m^*_c$ was evaluated from the slope.

In the first reports of the CR measurements in a 2DEG by photoconductivity, the effect was suppressed strongly by nonresonant photoconductivity effects, such as SdH oscillations, for example [91, 92], coming from a 2DEG heating by the THz radiation. The first observation of a CR as a single sharp peak was made by Maan et al. [93] on GaAs/GaAlAs samples of a high electron mobility.

The cyclotron resonance in photoconductivity is observed due to the change in the elec-
Figure III.1. Photocurrent spectra of samples P3 (a) and P4 (b) for laser lines 186 $\mu$m, 163 $\mu$m and 118.8 $\mu$m as the function of magnetic field. Numbers at cyclotron resonance peaks mark the wavelengths. Insets: symbols - position of the CR peak in $B$. Solid curves are linear fits of $\epsilon_c = \hbar e B/m^*_c$ from which $m^*_c$ for each sample were deduced.

electron distribution around the Fermi level. If the Fermi level lies between two successive Landau levels and $\hbar \omega_c \gg k_B T$, then the electron scattering from the fully occupied lower to the empty higher energy level is not efficient and the resistivity decreases. When photons of a resonant frequency are present, the electrons are photoexcited to the higher Landau level and a redistribution of electrons on states in the higher level as well as on empty states in the lower level induces changes in the scattering and the resistivity increases abruptly. On the
Table III. The radiation power measured at the laser output at different frequencies $\lambda_l$ during the experiments on samples P1, P2, P3 and P4.

<table>
<thead>
<tr>
<th>$\lambda_l$ [$\mu$m]</th>
<th>Sample</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.5</td>
<td></td>
<td>12 mW</td>
<td>10 mW</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>118.8</td>
<td></td>
<td>20 mW</td>
<td>22 mW</td>
<td>35 mW</td>
<td>32 mW</td>
</tr>
<tr>
<td>163</td>
<td></td>
<td>-</td>
<td>-</td>
<td>16 mW</td>
<td>17 mW</td>
</tr>
<tr>
<td>186</td>
<td></td>
<td>-</td>
<td>-</td>
<td>6 mW</td>
<td>5.5 mW</td>
</tr>
</tbody>
</table>

other hand, when the Fermi level is in the middle of a partially filled Landau level, a relative change in resistivity is weaker, as a part of electrons is thermally excited to the empty states above Fermi level in the same Landau level. Magnitude of the resonant photosignal appears to be directly proportional to the resistance dependence on magnetic field.

Coming back to the panel (a) in Fig. (III.1) and comparing the magnitudes of CR peaks, a non-monotonic scaling with the laser power is seen (an average laser power at each laser frequency used is given in Table III). We believe that this effect results from the different electron distribution at the Landau levels in magnetic field, where the CR peak is observed for a given laser frequency. An illustrative example is the magnitude of the cyclotron resonance peak at $\lambda_l = 163$ $\mu$m wavelength which is significantly higher than the magnitude of the peak at $\lambda_l = 118.8$ $\mu$m, despite of the fact that the laser output power of the latter line was more than twice as high as the power of the former one (see Table III). Also, the laser output power at $\lambda_l = 118.8$ $\mu$m was almost six times higher than at $\lambda_l = 186$ $\mu$m, while for the sample P3 the magnitude of the CR is larger by a factor of two, respectively.

We would like to attribute the following observation to the effect of the sample resistance change during a sweep of the magnetic field which was briefly introduced previously. In Fig. (III.2) magnetotransport curves for the samples P3 and P4 are plotted as a function of $B$ in red and green, respectively. Arrows mark the position of the CR for each spectrum taken from Fig. (III.1). Numbers at the arrows are the wavelength of the exciting laser line expressed in microns. As we can see, in the case of the sample P3, the CR for the laser line at 163 $\mu$m is the closest to a resistance minimum, where a relative change in resistance is the largest according to the predictions of the photoconductivity model. The change in the photocurrent is directly proportional to a relative change in resistance, thus the photocurrent
Figure III.2. The magnetoresistance spectra of samples P3 (red curve) and P4 (green curve). The arrows mark CR positions for laser wavelengths given by the numbers adjacent to arrows in the units of microns.

must change the most strongly here. Other CR peaks are closer to local resistance maxima where a weaker photocurrent is expected.

For the sample P4, the strongest CR response is at the wavelength of $\lambda_l = 118.8 \, \mu m$. This is an expected result as in this case the CR occurs when the Fermi level is between Landau levels and also the laser line is the most powerful. As for the weaker lines at lower frequencies they are close to local maxima in resistance, thus their magnitudes are much smaller. Also, the fact that at $\lambda_l = 163 \, \mu m$, a CR peak is stronger for P3 sample than for P4 sample and vice versa in case of $\lambda_l = 118.8 \, \mu m$, are consistent with our interpretation. It is difficult to compare the magnitudes of the CR at $\lambda_l = 186 \, \mu m$ for both samples, as $B_c$ in this case is on a shoulder of resistance maxima, approximately equally spaced from the extrema.

Unfortunately, we cannot apply the proposed models to explain measured results only qualitatively, as we do not know a spectral absorption of the samples and the polarization of THz radiation after propagation through the waveguide between laser and the sample. Also, a spatial distribution of the laser intensity on the sample surface is unknown.

Nevertheless, the observed single sharp CR peaks at practically zero background suggests
a potential use of investigated samples as $B$–tunable THz detectors.

B. Quantum oscillations in a terahertz photocurrent

The photocurrent spectra recorded with the chopped monochromatic laser radiation of wavelengths of 118.8 $\mu$m and 96.5 $\mu$m are shown in Fig. (III.3) (a) and (b) for the samples P1 and P2 respectively. The samples were biased with a constant drain-source voltage $V_{ds} = 20$ mV. The experimental photocurrent curves demonstrate an oscillatory behaviour, something very different from observations on the samples P3 and P4 discussed in the previous subsection. The peak resulting from the cyclotron resonance transition for given photon energy is observed in every spectrum and is denoted by “CR”. Before evaluating the cyclotron effective mass one needs to take into account that the laser frequencies used to excite the specimen are quite close to the LO phonon frequency $f_{LO} = 5.08$ THz, and the cyclotron effective mass increases due to the resonant polaron effect. Therefore, corrections to the CR frequency were introduced in each case according to the dependence $\omega_c(B)$ obtained for a 20-nm-wide CdTe/CdMgTe QW in Sec. II A. The cyclotron effective mass including the correction to compensate for the resonant polaron effect was evaluated to be $m^*_c \approx 0.102$ [see the insets in Fig. (III.3) (a) and (b)].

The nature of a cyclotron resonance peak in photocurrent was described in the previous chapter, however, here we have observed a photocurrent of a negative sign at the cyclotron resonance. The negative CR in photoresponse was observed earlier [94–97] and in general it depends on the magnetic field (LLs filling factor), on the bias voltage, on the wavelength and on the intensity of a THz light [96]. Thiele et al. [95] have found that in a photovoltage excited in a 2DEG based on GaAs by a THz radiation, the CR changes its sign around an even filling factors, and have explained this behaviour by the shift in electrochemical potential governed by a resonant heating of a 2DEG.

Now, let us concentrate on the observed photocurrent oscillations in magnetic field. Comparing these oscillations measured on the same sample but at different radiation frequencies a dependence of their magnitude on radiation power is seen. The oscillation period, however, does not change in any manner. This implies that we are dealing with a photocurrent arising due to non-resonant excitations. The oscillations were found to have a constant period in a reciprocal magnetic field. Also, it was observed that in strong magnetic fields, the period
Figure III.3. Photocurrent spectra of samples P1 (a) and P2 (b) for laser lines 118.8 µm and 96.5 µm as the function of magnetic field showing the Shubnikov-de Haas effect. Spectra representing different laser wavelengths are shifted vertically for clarity. Numbers at the curves mark the wavelengths. Insets: symbols – position of CR peak in $B$. The position was corrected according to Fig. (II.2) in order to minimize an influence of the resonant polaron effect on $m^*_c$. Solid curves are linear fits of $E_c = \hbar eB/m^*_c$ with interception at zero, from which $m^*_c$ was deduced.

These phenomena reveal a close similarity of the oscillating photocurrent to the Shubnikov-de Haas effect in magnetotransport, where the magnetoresistance oscillates periodically in $1/B$ and the oscillation frequency doubles when
Landau levels are spin resolved. The following gives us a clue to interpret a photocurrent dependence on the quantized electron density of states.

To explain the oscillating photoresponse in magnetic fields, a model of a 2DEG heating described in detail in Ref. [98] by Neppl et al. is used. The oscillations seen in the photocurrent are caused by a non-resonant heating of the electron gas (a bolometric effect) inducing an oscillating conductivity $\Delta\sigma$ of the electron plasma in $B$ domain. The change in conductivity is directly proportional to the difference in the electron and lattice temperatures $\Delta\sigma \propto \Delta T = T_e - T_l$. Of course, this is a very crude bolometer model, as it does not take into account lattice heating, nor a rate of the heat dissipation to a heat sink. A more explicit description considers a sample in a cryostat as the bolometer comprised of three parts connected via heat sinks: a 2D electron layer, a crystal lattice and a helium bath which are in the thermal equilibrium when the external radiation is absent. When the incoming radiation excites a single electron, it is scattered elastically either by other electrons or by the lattice and through this act transfers an excess kinetic energy gained from the excitation. Considering the electron-electron scattering as dominant, due to the relaxation of energy of many excited electrons, the 2DEG gains the temperature $\Delta T_e$ with respect to the lattice. The temperature of a hot electron sheet will relax to the temperature of the crystal lattice in a time $\tau_e$. In the next energy exchange cascade, the lattice temperature $\Delta T_l$ is dissipated to the helium bath, and this dissipation takes place in time $\tau_l$. In general, the electron conductivity change caused by absorbed THz radiation will be influenced by the 2DEG heating and the lattice heating. Thus, the following equation combining both effects describes $\Delta\sigma$ [98]:

$$\Delta\sigma = \frac{\partial\sigma}{\partial T_e} \Delta T_e + \frac{\partial\sigma}{\partial T_l} \Delta T_l = \frac{\partial\sigma}{\partial T_e} P\tau_e C_e + \frac{\partial\sigma}{\partial T_l} P\tau_l C_l. \quad \text{(III.1)}$$

Here $C_e$ and $C_l$ stand for the 2DEG specific heat and lattice specific heat, respectively, while $P$ is the absorbed power of a far-infrared electromagnetic wave. The ratio $\Delta T_e/\Delta T_l \approx 400$ at $T = 1.4$ K was found for Si-inversion layers with the mobilities of a several tens of thousands cm$^2$/Vs [98]. As the lattice heating is significantly smaller than that of the electron plasma, the second term at the right-hand side of the Eq. (III.1) can be neglected, therefore a change in conductivity $\Delta\sigma$ resulting from heating is mainly influenced by a bolometric effect in the 2DEG:

$$\Delta\sigma = \frac{\partial\sigma}{\partial T_e} P\tau_e C_e. \quad \text{(III.2)}$$
In general, the electron specific heat $C_e$ at the constant temperature is:

$$C_e = \int_0^\infty \frac{df}{dT}(\epsilon - \epsilon_F)D(\epsilon_F)d\epsilon.$$

(III.3)

with $D(\epsilon_F)$ denoting a density of states (DOS), $f$ standing for a Fermi distribution function and $\epsilon_F$ being the Fermi energy. Quantity $df/dT$ is given by:

$$\frac{df}{dT} = -\frac{\partial\epsilon}{\partial T} \left( \frac{\epsilon - \epsilon_F}{T} \frac{\partial\epsilon_F}{\partial T} \right).$$

(III.4)

The integral expression given for the specific heat in Eq. (III.3) can be reduced to an analytical form under the following assumptions. For cryogenic temperatures experiment conditions and 2DEG concentrations $n_s \gtrsim 10^{11}$ cm$^{-2}$, the condition $k_B T \ll \epsilon_F$ is satisfied. Also, the DOS must be approximately constant in the scale of $k_B T$, which is especially true for metals, but is not obvious for an electron gas with magnetic field induced energy quantization. In our specific case ($T = 1.8$ K), a thermal broadening $\Delta E_{th} = k_B T \approx 155$ $\mu$eV, and a broadening of Landau levels $\Gamma \approx 600$ $\mu$eV (see subsection IA.1). Thus, we believe that in our case a classical Sommerfeld relation (see for example Refs. [99, 100]) can be used to express the specific heat of a two-dimensional electron gas with a quantized energy:

$$C_e = \pi^2 k_B^2 T D(\epsilon_F)/3.$$

(III.5)

Strictly speaking, to apply Eq. (III.5), the condition $\Delta E_{th} \ll \Gamma$ must be satisfied. However, as long as we are not pretending to explain our experimental results quantitatively, this expression is very handy to demonstrate that $C_e$ is a linear function of the DOS.

Having the latter in mind, let us try to construct a picture, which would help to understand the oscillatory behaviour of the photocurrent when the magnetic field is changed. In quantizing magnetic fields, electronic states are quantized to Landau levels, and at zero or close-to zero temperatures, a density of states $D(\epsilon)$ is dependent on the field and shows a form of a series of narrow peaks. Ideally, $D(\epsilon_F, B)$ contains a series of $\delta$-spikes, but as the broadening takes place, spikes are expected to gain a Gaussian or Lorentzian shape. If, for example, the density of states $D(\epsilon)$ is assumed to be the sum of Gaussian peaks [101], then

$$D(\epsilon) = \frac{1}{(\pi k_B)^2 \Gamma} \sum_{n=0}^{\infty} \sqrt{\frac{2}{\pi}} \exp \left[ -\left( \frac{\epsilon - \epsilon_n}{2 \ln 2\Gamma} \right)^2 \right],$$

(III.6)
where $l_B = \sqrt{\hbar/eB}$ is the magnetic length, $\lambda_n = \hbar e B/m^*_e (n + 1/2)$ is the Landau energy and $\Gamma$ is the width (FWHM) of a broadened Landau level.

According to Eqs. (III.2) and (III.5), when the Fermi level falls between two successive Landau levels, the specific heat drops, forming a local maximum in $\Delta \sigma(B)$. If the Fermi level occurs to be in the middle of a broadened Landau level, where $C_e$ is the largest, a local minimum of $\Delta \sigma(B)$ is reached. The oscillating electron specific heat was demonstrated in a 2DEG subject to quantizing magnetic fields both theoretically [101] and experimentally [102]. Equation (III.2) clearly shows that the conductivity changes due to an electron plasma heating oscillations in the magnetic field. Furthermore, as it depends on the electron density of states, the oscillations are periodic in the inverse of $B$ – the effect similar to the Shubnikov-de Haas effect in magnetotransport. In strong magnetic fields, where Landau levels are spin resolved, oscillations double their frequency and develop for odd filling factors also as it can be seen from Fig. (III.3).

On the other hand, a temperature increase of the 2DEG also changes the electron Fermi-Dirac distribution, what in turn alters the scattering efficiency and thus the photocurrent. In this case, a difference between the current in the dark $I_d$ and the current in the light $I_l$ is the measured photocurrent $I_p = I_l - I_d$. The lowest scattering efficiency occurs when an integer number of the Landau levels are occupied, because in the vicinity of $\epsilon_F$ there are no free electron energy states to scatter to. At the resonant condition, when $\omega = \omega_c$ the photoresponse increases following the increased scattering because of electron excitation to an empty states above $\epsilon_F$ and unoccupied states left beneath $\epsilon_F$.

Unfortunately, our experiments cannot distinguish between the mechanisms responsible for a non-resonant bolometric effect in the samples due to the THz illumination. Also, one can not rule out a possibility that both of the mechanisms discussed (a non-resonant bolometric effect and electron redistribution due to the THz induced heating) contribute to the photoresponse of the investigated samples.

1. Double-peak structures in the quantum oscillations of the photocurrent

A closer study of the spectra in Fig. (III.3) reveals a split of a SdH maximum in photocurrent at $B \sim 6 - 7$ T. It cannot come from spin resolved Landau levels, because positions of two split peaks do not correspond to a period of the second harmonic of SdH oscillations.
In order to study the effect more thoroughly, the photocurrent spectra of the samples P1 and P2 at radiation frequencies of 96.5 \( \mu \text{m} \) and 118.8 \( \mu \text{m} \) were depicted as a function of Landau level filling factor in a Fig. (III.4) (a) and (b), respectively. A very clear splitting of the single peak to two well pronounced maxima is observed at \( \nu = 2 \). Also, a highly non-symmetric shape is seen for maxima at \( \nu = 3 \) and 4, together with bumps close to \( \nu = 1 \). Furthermore, we have observed splitting of this type in the high electron mobility CdMnTe/CdMgTe quantum wells (samples P5 and HB) as well [panels (c) and (d)]. The fact that this kind of splitting was not observed in the quantum oscillations in a DC transport measurements is very intriguing as well. A similar photoresponse behaviour was reported earlier in Corbino geometry samples based on III-V heterostructures such as GaAs/AlGaAs quantum wells with a high electron mobility [96, 103, 104] and InSb/AlInSb QWs [105], II-VI quantum wells of HgTe/HgCdTe [106] and also in graphene [107]. The mentioned reports conclude that double peaks in the photocurrent are observed whenever an integer quantum Hall effect manifests in magnetotransport.

As it can be seen from Fig. (III.4), the gap \( \Delta B \) between the two split peaks does not depend on radiation frequency, but rather varies from sample to sample. This suggests a non-resonant nature of the effect responsible for the splitting. To explain the observed phenomenon qualitatively, we will follow the approach suggested by Nachtwei et al. [104].

As it was mentioned above, an absorption of THz radiation in the sample increases the temperature \( T_e \) of the electron gas relative to the lattice, by the value \( \Delta T \). An increase in temperature changes the conductivity tensor and in this way alters the photocurrent. The electron system is extremely sensitive to a THz excitation in a quantum Hall regime. However, a raise in temperature \( T_e \) can cause a breakdown of the QH state to a dissipative state (for a review see [108] and references therein). Under the illumination with a chopped THz radiation, the energy absorbed by the electron gas during one radiation on-off cycle is:

\[
\Delta E = A C_e \Delta T,
\]

with \( A \) standing for an illuminated surface of the sample, \( \Delta T - 2\text{DEG} \) temperature increase. \( C_e \) is the specific heat of the electron gas, defined by Eq. (III.3), but with the electron density of states not at the Fermi level but rather as a function of the LLs filling factor. Substituting the expression of \( C_e \) to the Eq. (III.7), a temperature increase is written in the following form:
Figure III.4. The oscillating non-resonant photoresponse as a function of Landau level filling factor $\nu$ of the samples P1, P2, P5 and HB [panels (a), (b), (c) and (d), respectively]. Photocurrent was induced with FIR radiation of the wavelengths $118.8\,\mu m$ (blue curve) and $96.5\,\mu m$ (red curve) in every case. The simultaneously measured magnetotransport is shown by dashed green curve.
Figure III.5. Photocurrent spectra as the function of the Landau level filling factor of sample P5 measured for the FIR photon wavelength of 118.8 μm at temperatures of 1.8 and 4.2 K (upper and bottom curves, respectively).

\[
\Delta T(\nu) = \frac{3\Delta E}{\pi^2 k_B T D(\nu) A} = \frac{K}{D(\nu)}. \tag{III.8}
\]

Supposing that a THz radiation absorption would increase the electron temperature, the increased electron temperature can be expressed as the sum of the lattice temperature \( T_l \) and a temperature change due to absorption, \( \Delta T \):

\[
T_e = T_l + \Delta T. \tag{III.9}
\]

A temperature increase caused by the electron gas absorption, \( \Delta T \), is given by the Eq. (III.8). Now, from the previous discussion it is clear that \( \Delta T \) depends on the position of Fermi level, i. e. the filling factor of Landau levels. The enhancement of the temperature is the largest at an integer \( \nu \), where the DOS is the smallest.

The double peak structure observed at an integer filling factors is explained by breaking down of a QH state. The latter can be realized by increasing the temperature or a current density flowing through the sample [109, 110]. As our investigated CdTe/CdMgTe quantum wells are of a high electron mobility, we believe that the manifestation of breaking down of a QH state is the case here. To be exact, magnetotransport measurements do not show quantum Hall states in our samples, however, a two-terminal experiment geometry rules out the
Figure III.6. Model calculation of the electron temperature $T_{el}(\nu)$ and the critical temperature $T_{\text{crit}}(\nu)$ near $\nu = 2$ according to Eqs. III.9 and III.10 respectively. Parameters used in the calculations: $k_B T_{\text{lat}} = 0.2$ meV, $\nu = 2$ and $n_s = 2.42 \times 10^{11}$ cm$^{-2}$ (picture taken from [104]).

possibility to observe the QHE. On the other hand, our assumption is supported by magnetotransport experiments which we carried out on the Hall bar based on the CdTe/CdMgTe QWs with $L_{QW} = 20$ nm modulation doped at one barrier and with electron concentrations of a few $10^{11}$ cm$^{-2}$; the wafer nominally very similar to ours [32].

To explain the origin of the double peak structure, the elevated electron temperature $T_{e}$ is compared with the critical temperature $T_{\text{crit}}$, which is needed to destroy the QHE. $T_{\text{crit}}$ is chosen to be a periodic function of the filling factor, gaining the largest value at the integer filling factors, and decreasing when $\nu$ recedes from the integer value $\nu = n$ ($n = 1, 2, 3, ...$). A critical temperature function on $\nu$ was chosen to be of the form:

$$T_{\text{crit}}(\nu) = \frac{T_{\text{crit}}(n)D(n)}{D(\nu)}, \quad (\text{III.10})$$

where $T_{\text{crit}}(n)$ is the critical temperature and $D(n)$ is the density of states at the integer filling factor $n$.

The $T_{e}(\nu)$ and $T_{\text{crit}}(\nu)$ calculated by Nachwei et al. [108] for a high electron mobility
system with the aid of Eqs. (III.9) and (III.10) respectively, are compared (depicted as solid and dashed curves, respectively) in Fig. (III.6). A constant lattice temperature $T_l$ is represented by a double-dash-dotted horizontal line. As it is seen, there are regions where:

(a) $T_l < T_{\text{crit}}(\nu) < T_{e}(\nu)$; (b) $T_{\text{crit}}(\nu) > T_{e}(\nu) > T_l$ and (c) $T_{\text{crit}}(\nu) < T_l < T_{e}(\nu)$.

When the radiation is not present, the electrons are at the temperature $T_e = T_l$ (in the thermal equilibrium with a crystal lattice), and when $T_e < T_{\text{crit}}(\nu)$, the system is in quantum Hall state. Outside this interval [solid vertical lines in Fig. (III.6)] $T_e > T_{\text{crit}}(\nu)$ and 2DEG are in the dissipative mode.

If the sample is illuminated with THz waves, the electron temperature increases and intersects the curve of the critical temperature at the points marked by the dotted vertical lines. In the intervals between the solid and the dotted lines, the condition (a) is satisfied, QH state is broken and system is in the dissipative state. Thus, a bolometric photocurrent behaves according to the Eq. (III.2), that is it increases towards $\nu = 2$. But when $\nu$ approaches still closer to $n = 2$, it enters into the region limited by dotted lines, where $T_{\text{crit}}(\nu)$ exceeds $T_e(\nu)$ [condition (b)], and a 2DES is in the integer QH state. Photocurrent is small in a QH state [94] as electrons are resonantly excited mostly to a localized states. Therefore, at the point $T_{\text{crit}}(\nu) = T_e(\nu)$ photocurrent is maximum and becomes negligibly small when an integer filling value is reached.

With a further decrease of the filling factor, the photocurrent reaches local maximum at the second intersection point of $T_{\text{crit}}(\nu)$ and $T_e(\nu)$ and decreases in the region $T_{\text{crit}}(\nu) < T_e(\nu)$. For $\nu$ outside the interval enclosed by solid vertical lines, the amount of heat absorbed by the electron gas is not as high as at an integer filling factor, and the photocurrent signal weakens.

A discussed mechanism suggests the structure of a double peak in the photocurrent dependence of filling factor with a minimum exactly at an integer filling factor and two maxima at the flanks of a QH plateau, where the heating by the radiation induces the breakdown of a QH state. The model was constructed assuming the Landau level broadening $\Gamma$ independent of the magnetic field, and $D(\nu)$ calculated numerically at a lattice temperature instead of the one calculated at elevated electron temperature $T_e(\nu)$. On the other hand, this approach explains the essential physics beyond the observed splitting of the peak in photocurrent at integer filling factors.

However, the proposed model fails to explain further reduction of photocurrent toward the
maximum in SdH. Here, the bolometric photoresponse model which was discussed previously must be remembered, which predicts a reduction of the temperature increase $\Delta T$ at a maximum $D(\nu)$ at a half-filled Landau level.

C. Why the quantum oscillations of photocurrent are not observed in samples P3 and P4?

Comparing photocurrent spectra of two different set of samples in Figs. (III.1) and (III.3) the lack of quantum oscillations for the samples P3 and P4 is obvious. In our opinion, an overall temperature of the sample (of the electron gas and the lattice) is one of the most important factor responsible for this difference because of the following reasons:

(a) The helium bath temperature was 4.2 K and 1.8 K during the experiments on samples P1, P2 and P3, P4, respectively. In order to demonstrate the photocurrent dependence on the helium bath temperature, a photocurrent spectra of the sample P5 at $T = 1.8$ K and $T = 4.2$ K as a function of filling factor $\nu$ are shown in Fig. (III.5). The magnitude of the features arising from a non-resonant heating are of approximately two times lower in the case when $T = 4.2$ K. Also, the CR peak is lower by an order of magnitude for the same temperature.

(b) Shubnikov – de Haas oscillations arise in the photocurrent due to a non-resonant 2DEG heating by incoming THz radiation. Meanwhile, a cyclotron resonance peak in the photocurrent is the result of a carrier density function change due to a resonant electron excitation by photons to a higher successive Landau level. Both mechanisms might act simultaneously [91]. However, a non-resonant heating might be suppressed if the 2DEG is heated by the current passing through the sample (see [111], for example). Our experimental results suggest that a non-resonant heating mechanism is suppressed in the samples P3 and P4 and only a resonant excitation remains. We explain it by a 2DEG heating in samples P3 and P4 because of a higher bias voltage $V_{ds}$. During the experiment, $V_{ds} = 150$ mV and typical sample resistance at zero field $R_3 \approx 44$ k$\Omega$ and $R_4 \approx 95$ k$\Omega$ for samples P3 and P4 respectively, this would give the thermal (Joule) power $P_{t3} = V_{ds}^2/R_3 \approx 510$ nW and $P_{t4} \approx 235$ nW. For samples P1 and P2 ($V_{ds} = 20$ mV, $R_1 \approx 5.3$ k$\Omega$ and $R_2 \approx 6.2$ k$\Omega$) the thermal power $P_{t1} \approx 75$ nW and $P_{t2} \approx 65$ nW, respectively. From the simple calculations it is obvious that the bias voltage heats samples P3 and P4 at least three times stronger than
samples P1 and P2.
IV. MAGNETOPLASMONS IN A CDTE/CDMGTE QUANTUM WELLS

A. Magnetoplasmons in grid-gated samples

The transmission spectra measured at laser frequencies of 2.52 THz and 3.11 THz on the samples G1, G2 and GR are shown in Fig. (IV.1). Each spectrum presented is the result of averaging over 15 sweeps of magnetic field taken separately.

Figure IV.1. Transmission dependence on the magnetic field for samples, G1 [$\Lambda = 2 \mu m$, (a) and (b)] and G2 [$\Lambda = 3 \mu m$, (c) and (d)] at the laser frequencies of 2.52 THz and 3.11 THz. Triangles mark peaks of magnetoplasma resonances. Lorentzians resulting from the deconvolution procedure in (b) and (c) are represented by dotted lines.
A deep minimum visible in each spectrum corresponds to the cyclotron resonance (CR) transition. For the laser frequency of 2.52 THz a cyclotron effective mass \( m^*_c = (0.1008 \pm 0.0005) m_0 \) is determined for this 20 nm-thick CdTe quantum well where cyclotron resonance peak is observed at \( B \sim 9.1 \) T is in a good agreement with the value obtained by Karczewski et al. \[39\]. For laser frequency of 3.11 THz, the CR peak is at magnetic fields around 11.5 T, what gives a significantly higher effective mass \( m^*_c = (0.1029 \pm 0.0005) m_0 \). We believe that this increase in the cyclotron mass results from a resonant polaron effect which, for the same samples, is discussed in section II A and Ref. \[112\]. The increase in \( m^*_c \) becomes higher when \( \omega_c \) approaches \( \omega_{LO} \), what is clearly shown in Fig. (II.4). In order to account for the increase, we have found that in our case, the cyclotron effective mass can be approximated by a following second order polynomial function of magnetic field in the interval between 7 and 14 T:

\[
m^*_c(B) = (0.1102995 - 0.0025604B + 0.0001674B^2)m_0.
\] (IV.1)

Here \( m_0 \) is a free electron mass. As it will be shown later, this approximation introduces significant corrections in plasmon dispersions, especially at radiation frequencies close to LO phonon frequency.

A comparison of spectra in Fig. (IV.1) clearly shows a symmetrical CR line shape for the sample GR and a CR minimum broadened at a low-B shoulder, consisting of at least a few dips [their positions are marked with triangles in Fig. (IV.1)] in the case of samples G1 and G2, containing the metal grating on the top. We interpret them as features resulting from excitation of the fundamental and three subsequent two-dimensional magnetoplasmon modes in a free electron gas. We have found only one report in the literature on 2D magnetoplasmons in II-VI materials (namely in Cd(Mn)Te/CdMgTe QWs), which were observed in Raman spectroscopy experiment \[41\], where it was found out that the dispersion did not follow a square root law of the wavevector. The use of a far-infrared radiation allows to observe of plasmon resonances directly, because a typical energy of radiation is of the same order of magnitude as the energy of plasmons. On the other hand, in order to couple to plasma oscillations, momentum conservation must be satisfied as well. In general, the magnitude of photon momentum which can be transferred in a direction parallel to the incident surface relies on the angle of incidence \( \gamma \): \( p = h\omega/c \sin \gamma \), where \( \omega \) is radiation frequency and \( c \) is light velocity; what means that at very small \( \gamma \), the longitudinal momentum component carried by the THz light is negligible. Thus, THz radiation with the wavevector normal to a
2DEG sheet (Faraday configuration of the experiment, for example) does not carry enough momentum to couple to 2D plasmon, i.e., it cannot excite plasma oscillations. However, 2D plasmons are dispersive excitations and it is possible to meet the requirement of momentum conservation at certain wavevectors by placing periodic metal grating on the top of the sample. Then an electromagnetic wave diffracts on the grating and its electric field component in the near field is modulated by the grating period. Additional electric field modulation results in a gain in the momentum of incident light \( \hbar \Delta k_j = \hbar 2\pi j / \Lambda \), which is quantized at \( j = 1, 2, 3, ... \) and defined by the grating period \( \Lambda \). Since the phase velocity of plasmon is much smaller than \( c \), the initial (before diffraction) longitudinal momentum is neglected and plasmon wavevector \( k_{p,j} \) is equal to \( \Delta k_j \). This reasoning is illustrated by Fig. IV.2, where plasmon dispersion (solid curve) is depicted together with the dispersion of light, diffracted on the grid (dotted curves). Plasmon dispersion was calculated assuming that a metal grid totally screens 2DEG and taking typical parameters of our samples: \( n = 3 \times 10^{11} \text{ cm}^{-2} \), and \( \varepsilon_s \sim \varepsilon_b = 10.8 \). The curve representing it crosses light dispersion curves at wavevectors \( k_{p,j} \), i.e. only plasma oscillations with these wavevectors can be induced. The concept of a metallic grid as an efficient photon-plasmon coupler was used in a number of experiments where plasmon excitations were observed \cite{65, 113-116}.

We expect that magnetoplasmons observed in transmission experiments on grid-gated
samples are described by the wavevector \( k_{p,j} = 2\pi j/\Lambda \).

In order to find a full width at half maximum (FWHM) of magnetoplasmon and CR peaks, the baseline was subtracted and the procedure of deconvolution to Lorentzian peaks was performed on the spectra shown in Fig. (IV.1) (b) (sample G1) and (c) (sample G2). The results are marked by dotted lines. An average FWHM of magnetoplasmon peaks [0.10 T and 0.16 T for samples G1 and G2, respectively] was found to be very close to that of the CR peak (0.10 T and 0.14 T) which suggests that, in general, they are approximately equal to each other. The width of resonances in sample G2 is higher by approximately 50% than in sample G1. This could result from different patterns of scattering centers in these two samples cooled separately in different experimental runs.

1. **Plasmon-LO phonon interaction**

As it was discussed previously, CdTe is a highly polar material with a relatively low optical phonon energy (\( \omega_{\text{TO}} = 2\pi \times 4.2 \text{ THz} \) and \( \omega_{\text{LO}} = 2\pi \times 5.1 \text{ THz} \)). It was already demonstrated in the FTIR transmission experiment that at the radiation frequencies we have used (which are quite close to optical phonon frequencies in CdTe), the cyclotron mass increases due to the resonant polaron effect. Therefore, contribution coming from plasmon interaction with optical phonons must be taken into account in plasmon and magnetoplasmon dispersion relations. As collective electron movement in electron plasma has a longitudinal nature, plasmon can only couple to LO phonon, therefore we will refer to LO phonon simply as phonon further in this chapter. The interaction of the phonon frequencies comes through a screening of 2DEG, i.e. modified dielectric constant of the material.

The dynamical properties of the lattice are introduced simply by replacing the static dielectric constant of the lattice \( \varepsilon \) with a dielectric function that depends on frequency [117, 118]:

\[
\varepsilon(\omega) = \varepsilon_{\infty} \frac{\omega^2 - \omega_{\text{LO}}^2}{\omega^2 - \omega_{\text{TO}}^2}.
\]

\( \varepsilon_{\infty} \) is a high frequency dielectric function, related to a static dielectric constant \( \varepsilon \) by a Lyddane-Sachs-Teller relation \( \omega_{\text{LO}}^2 = (\varepsilon_0/\varepsilon_{\infty})\omega_{\text{TO}}^2 \) [119]. The influence of plasmon-phonon coupling on magnetoplasmon frequency is introduced in the term \( \omega_p^2(k, d) \) in Eq. V.15. Here, static dielectric constants \( \varepsilon_a \) and \( \varepsilon_b \) in the effective dielectric function \( \bar{\varepsilon}(k, d) \) are replaced with
frequency functions \( \varepsilon_s(\omega) \) and \( \varepsilon_b(\omega) \) defined by Eq. (IV.2), respectively. If we assume gated plasmons, in the expression given by Eq. (V.11) the common term \( (\omega^2 - \omega_{LO}^2)/(\omega^2 - \omega_{TO}^2) \) can be moved in front of the parentheses leaving only high frequency dielectric constants instead of their static counterparts:

\[
\bar{\varepsilon}_g(\omega, k_j, d) = \frac{1}{2} \frac{\omega^2 - \omega_{LO}^2}{\omega^2 - \omega_{TO}^2} \left[ \varepsilon_{s,\infty} + \varepsilon_{b,\infty} \coth(k_j d) \right].
\] (IV.3)

However, due to a complicated dependence on \( \varepsilon_b \), a similar algebraic manipulation cannot be done in case of ungated plasmons with an effective dielectric function defined by Eq. (V.12). However, under the condition \( \varepsilon_b \gg 1 \), the dielectric function for ungated plasmons can be approximated to the same form as \( \bar{\varepsilon}_g(k_j, d) \) and the frequency term moved to the beginning at right hand side:

\[
\bar{\varepsilon}_{ug}(\omega, k_j, d) = \frac{1}{2} \frac{\omega^2 - \omega_{LO}^2}{\omega^2 - \omega_{TO}^2} \left[ \varepsilon_{s,\infty} + \varepsilon_{b,\infty} \tanh(k_j d) \right].
\] (IV.4)

Considering \( \varepsilon_b \approx 10 \) (a typical value for a CdTe-based semiconductor) such an approximation gives an error within 2% for the highest plasmon harmonics observed (4th and 12th) in grid-gated and QPC samples, respectively. After putting Eq. (IV.2) into the denominator of \( \omega_p^2(k, d) \), substituting the resulting expression to Eq. (V.15) and some trivial mathematical manipulations we arrive to:

\[
\frac{\omega^2 - \omega_{LO}^2}{\omega^2 - \omega_{TO}^2} - \frac{\omega_{p,j}^2}{X_j^2} \sum_{l=1}^{\infty} \frac{4l^2 J_l^2(X_j)}{\omega^2 - (l\omega_c)^2} = 0.
\] (IV.5)

In conclusion, to include plasmon-LO phonon coupling in the two-dimensional magnetoplasmon dispersion within a non-local model, a few modifications to Eq. (V.15) are necessary:

1. The first term is replaced with \( (\omega^2 - \omega_{LO}^2)/(\omega^2 - \omega_{TO}^2) \).

2. Static dielectric constants which are encoded in the plasma frequency \( \omega_p^2 \) must be replaced with their high frequency counterparts.

However, the equation is not very convenient for a calculation. For a small or large \( X_j \) (at high or low magnetic fields or for plasma oscillations of sufficiently long wavelengths) a local approximation of Eq. (IV.5) can be used:

\[
\frac{\omega^2 - \omega_{LO}^2}{\omega^2 - \omega_{TO}^2} - \frac{\omega_{p,j}^2}{\omega^2 - \omega_c^2} \simeq 0.
\] (IV.6)
The range of its validity is material-dependent and can only be determined by solving Eq. (IV.5) to find a range of $X_j$ where non-local effects are negligible (see below). In Fig. (IV.3) the magnetoplasmon dispersion as a function of magnetic field is demonstrated for grid-gate periods of 2 $\mu$m and 3 $\mu$m (samples G1 and G2, respectively). The points denote magnetoplasmon resonances positions in the magnetic field taken from the experimentally recorded transmission spectra in Fig. (IV.1).

Dotted curves represent theoretical magnetoplasmon dispersion of the first four harmonics calculated in a local approximation with a help of Eq. (IV.6), which includes plasmon-phonon interaction. Solid curves are theoretical dispersion calculated in the same conditions in the frame of non-local model represented by the Eq. (IV.5). Solid red lines are the fundamental and two subsequent harmonics of cyclotron frequency. In this case, an infinite number of terms in the sum were limited to the first three. Using more terms of the sum would result in a more accurate dispersion relation reconstruction at low magnetic fields ($B < 0.7$ T). However, we are dealing with magnetoplasmons excited in a range of much stronger fields were limiting the series to the first three terms does not influence the accuracy of estimation.

Equation (IV.5) has a singularity at frequencies equal to $\omega_{TO}$ and $l\omega_c$, where $l = 1, 2, 3, \ldots$. Therefore, magnetoplasmon frequency in the non-local approximation is in anticross with the fundamental and higher harmonics of the cyclotron frequency and TO phonon frequency. Equation (IV.6) which describes magnetoplasma frequency within local model, has the singularity only at fundamental cyclotron frequency $\omega_c$ and TO phonon frequency $\omega_{TO}$. In Fig. (IV.3) in the region of $B < 3.5$ T an anticrossing of the cyclotron frequency harmonics of higher order with magnetoplasmon frequency harmonics, calculated according to a non-local model, is clearly observed. Meanwhile, magnetoplasmon frequency, calculated within a local model do not anticross with the higher harmonics of $\omega_c$. In both non-local and local model cases $\omega_p$ has to be in anticross with $\omega_{TO}$, but the energy scale in Fig. (IV.3) is too low to show it.

Comparison of magnetoplasma frequencies shows that in magnetic fields higher than 5 T the frequency obtained using local and non-local approximations are the same for the first four magnetoplasmons harmonics in a 20-nm-wide CdTe/CdMgTe QW. Therefore, in our case the range of magnetic field where magnetoplasmon resonances were observed ($8$ T $< B < 11.3$ T) it is safe to use Eq. (IV.6) in calculation of the plasma oscillation frequency.
Figure IV.3. Theoretical magnetoplasmon frequency of the main and three higher harmonics calculated for the samples G1 (a) and G2 (b) in the local (green dotted curves) and non-local (blue solid curves) approximation as a function of magnetic field. Red solid lines are the fundamental, the second and the third harmonics of the cyclotron frequency. Black points represent the experimental positions of magnetoplasma resonances at laser frequencies of 2.52 and 3.11 THz in $B$ domain.
For the highest mode \((j = 4)\) observed at 2.52 THz, the calculated values of the non-local parameter \(X_4\) are 0.14 and 0.11 (0.11 and 0.07 at 3.11 THz) for the samples G1 and G2, respectively. As can be shown using Eq. (IV.5), the value of \(X_4\) below which no significant influence of non-local effects on the dispersion relation should be seen, are equal to about 0.28 for G1 and about 0.19 for G2. This explains why the experimental results obtained on both grid-gated structures can be described by Eq. (IV.6), i.e., in a local approximation.

In plasma oscillation frequency estimation, a proper choice of an effective dielectric function plays an important role also. To our understanding, in the case of a metallic grating deposited on the top of the sample, the manner in which plasmons are screened should be intermediate between the situation of ungated plasmons, and plasmons gated by a continuous metallic plane: the two situations which were discussed previously. In general, gated plasmons localized under metallic fingers of the grid form mixed modes with ungated plasmons existing in the grid's openings. It has been recently shown for a GaN/AlGaN [70] and GaAs/AlGaAs [120] heterostructures that a very good phenomenological description of these modes can be obtained with an effective dielectric function approximated by a weighted average of effective dielectric functions for the gated, \(\bar{\varepsilon}_g\), and ungated, \(\bar{\varepsilon}_{ug}\), cases:

\[
\bar{\varepsilon} = (1 - \eta)\bar{\varepsilon}_g + \eta\bar{\varepsilon}_{ug},
\]

where \(0 \leq \eta \leq 1\) is the weighting parameter. For a grid coupler the parameter \(\alpha\) should be equal to the aspect ratio of the grating, which is defined as \(W/L\), where \(W\) and \(L\) are proportions of the gated and ungated parts of the sample.

Figure (IV.4) (a) and (b) shows plasmon frequency dependence on a normalized wavenumber for samples G1 and G2 (grating periods \(\Lambda = 2\) and 3 \(\mu m\), respectively) determined from the experimental spectra given in Fig. (IV.1). The wavenumber \(k_1\), to which the normalization was done, is the wave vector of the first plasmon mode.

As the discussion above implies, plasma oscillation frequency was calculated using the local approximation. The plasmon wavenumber was defined by the inverse of grating period \(\Lambda\), i.e., \(k_j = 2\pi j/\Lambda\), with \(j = 1, 2, 3, \ldots\). Plasmon dispersion is shown in the domain of wavenumber normalized to \(k_1 = 2\pi/\Lambda\). The cyclotron frequency \(\omega_c\) was estimated at the magnetic field defined by the position of magnetoplasmon resonances and \(\omega\) was taken to be equal to the laser energy. The obtained plasmon frequency \(f_p = \omega_p/2\pi\) at the laser frequencies of 2.52 and 3.11 THz are denoted by the squares and stars, respectively. Solid
Figure IV.4. Experimental plasmon dispersion for \( \Lambda = 2 \mu m \) (a) and \( \Lambda = 3 \mu m \) (b), at 2.52 THz (squares) and 3.11 THz (stars). Theoretical plasmon dispersion [Eq. (V.10)] with (solid line) and without (dotted line) plasmon-LO phonon interaction. Wavevector was normalized to the wavevector of the first plasmon mode.

lines are fits of Eq. (V.10) with an effective dielectric function described by Eq. (IV.7) and with fitting parameters: \( \eta = 0.45 \) (close to the geometrical aspect ratio of the grid) and \( \varepsilon_{b,\infty} = 5.9 \) (below the value of a high-frequency dielectric constant of bulk CdTe, \( \varepsilon_{\infty} = 7.1 \); this lower value can result from a complex structure of the barrier that comprises, in particular, an SiO\(_2\) layer, with its \( \varepsilon = 3.9 \). Dotted curves in Fig. (IV.1) (b) and (c) represent plasmon dispersion when the plasmon-LO phonon interaction is neglected, calculated according to Eqs. (V.10) and (IV.7), with \( \eta = 0.45 \) and using the values of static dielectric constants \( \varepsilon_s \) and \( \varepsilon_b \) to evaluate \( \bar{\varepsilon}_g(k_j, d) \) and \( \bar{\varepsilon}_{ug}(k_j, d) \). The value of \( \eta \) highlights a mixed nature of excited magnetoplasma waves, which – propagating through both gated and ungated regions of a 2D electron gas – are screened by the grid in a complicated manner.

As it is seen from the fit results, the interaction of plasmon and phonon increases the plasmon frequency by around 1.33 (1.45) times when excited by a far-infrared photon of 2.52 THz (3.11 THz). An increase in plasma oscillations frequency by a factor of \( (\omega^2 - \omega_{\text{LO}}^2)/(\omega^2 - \omega_{\text{TO}}^2) \) can be predicted by comparing Eqs. (V.16) and (IV.6). For the experimental frequencies of 2.52 THz and 3.11 THz, the theoretical increase in plasmon frequency was found to be 1.31.
Figure IV.5. Plasmon dispersion of the sample G1 calculated with a help of Eq. (IV.6) for magnetoplasmon resonances shown in (a) and (b) panels in Fig. (IV.1) excited with radiation of frequency of 2.52 THz (a), and 3.11 THz (b). Squares represent the results with \( m_c(B) \) defined by Eq. (IV.1), triangles - with a constant mass defined from CR position \( m_c = (0.1008 \pm 0.0005)m_0 \). The plasmon wavevector was normalized to a wavevector of the fundamental plasmon frequency harmonic.

and 1.42 respectively.

We would like to demonstrate how the resonant polaron effect caused increase in a cyclotron mass affects the plasmon dispersion. The change in \( m^*_c \) becomes important especially at the radiation frequencies close to \( \omega_{LO} \). The plasmon frequency was estimated for a sample G1 (grid period of 2 \( \mu \)m), for magnetoplasma resonances detected in spectra shown in Fig. (IV.1) (a) and (b) at photon frequencies of 2.52 THz [Fig. (IV.5) (a)] and 3.11 THz [Fig. (IV.5) (b)] by the procedure described before. Squares mark plasma frequencies obtained from the calculations taking electron cyclotron effective mass as a function of magnetic field \( m^*_c(B) \), which is described by Eq. (IV.1). Triangles are plasmon dispersion calculated for constant electron effective mass, defined by the position of the CR peak \( [m^*_c = (0.1008 \pm 0.0005)m_0] \). The wavevector \( k \) was normalized to \( k_1 \) - a wavevector of the fundamental harmonic of the plasmon frequency.

In the case of 3.11 THz, the disagreement in plasmon frequencies calculated with magnetic field dependent \( m^*_c(B) \) and with constant \( m^*_c \) is clearly visible. It becomes smaller when
the index of plasmon mode is increased. This results from the fact that magnetoplasmon resonances of a higher order occur at lower magnetic fields, where the increase of $m^*_c$ due to the resonant polaron effect is smaller.

In the case of 2.52 THz [Fig. (IV.5) (a)], an effect of the change in the effective cyclotron mass caused by the resonant polaron effect is negligibly small – the plasmon frequency calculated under the influence of the resonant polaron effect is practically the same as that when the influence mentioned is neglected.

B. Magnetoplasmons in the quantum point contact sample

Photocurrent spectra $P = P(B)$ of the QPC excited by the THz laser at the frequency equal to 2.52 THz are shown in Fig. (IV.6) (a) for a few different voltages $V_g$ that were symmetrically applied to the lateral gates. For convenience the spectra are shifted vertically. In all spectra, a broad structure with additional modulation is observed. However, neither a change of a peak center position, nor the modulation periodicity change was observed with a gate voltage.

As it is shown it is possible to deconvolve the broad spectral feature to a separate Lorentzian peaks (dotted curves). The peaks at a high field shoulder of the broad structure were added to account for different levels of a baseline at the low $-B$ shoulder and a high $-B$ shoulder of the structure. The value of the effective mass found from the position of the CR maxima is equal to $(0.1028 \pm 0.0005)m_0$ in a 15 nm-wide CdTe/CdMgTe quantum well and agrees with an estimation given in Ref. [39]. A series of spectral features accompanying the CR peak represents magnetoplasmon modes starting from the fundamental one up to its 12th harmonics. After a deconvolution procedure, a FHWM of the CR peak and magnetoplasmon peaks was found to be the same and equal to 0.32 T, much larger than that found for grid-gated samples [Fig. (IV.1) (a)]. We think that it is the disorder at etched mesa borders that results in this broadening since in the case of the QPC magnetoplasmon are defined by the mesa width.

A set of plasmon frequencies derived from their magnetic-field positions is displayed with open circles in Fig. (IV.6) (b). Drawn with a solid line is the dispersion for ungated plasma oscillations confined in the channel of the device that was fitted to the experimental data using the same approach as in the case of the grid-gated samples but with the wavelength
Figure IV.6. (a): Photocurrent signal recorded on the QPC at different $V_g$. Spectra are shifted vertically for clarity. Dotted vertical lines are guides to an eye to mark positions of maxima in the spectra. The cyclotron resonance position is denoted by CR. Deconvolution to Lorentzians is shown (dotted curves) for spectrum measured at $V_g = 0$ V. (b): Experimental frequencies of plasmon modes (open circles) fitted with a theoretical dispersion relation including plasmon-LO phonon interaction (solid curve) and neglecting it (dotted curve).

of the fundamental mode equal to the channel width $W = 2.4 \, \mu m$. The obtained fitting parameters are $\eta = 0.95$ and $\varepsilon_{b,\infty} = 6.3$. Here, the magnetoplasmon resonances are practically ungated and a residual screening they experience most probably results from the presence of lateral gates. Here, the local approximation [ Eq. (IV.6)] provides a good description of the plasmon dispersion, too [Fig. (IV.7)], even though the parameter $X$ reaches values as high
Figure IV.7. Theoretical magnetoplasmon frequency of first twelve harmonics calculated for the QPC sample in the non-local (blue solid curves) and local (green dashed curves) approximation as a function of magnetic field. Red solid lines are the fundamental and the second and the third harmonics of the cyclotron frequency. Black points represent the experimental positions of magnetoplasma resonances at laser frequencies of 2.52 THz in $B$ domain.

as 0.25 for the highest mode. According to the estimation based on Eq. (IV.5), however, at radiation frequency of 2.52 THz, the dispersion relation of magnetoplasma waves in the QPC should not be affected by non-local effects at $B > 5\,\text{T}$, what for the case of 12\text{th} harmonic of the fundamental mode means $X_{12} = 0.32$.

A theory of plasmons in an ungated 2DEG stripe with a constant width $W$ predicts the excitation of waves with a $k$-vectors commensurating a half-integer of a stripe width, i. e., both even and odd harmonics of the plasmons exist in such a system [121]. On the other hand, the excitation of odd harmonics only was reported [122]. In our analysis of experimental data, a plasmon selection rule $k_j = \pi j/W$ ($j = 1, 2, 3, \ldots$) is assumed, which allows for even and odd harmonics of plasma oscillations is assumed. Otherwise (i. e. to
keep odd or even harmonics only), a significant contribution of gated plasmons would have to be taken into account to explain our experimental plasmon dispersion. Due to a lack of the gate metalization, there is no evident reason for gated plasmons to be excited.

By applying a lateral gate voltage to the QPC it should be possible to reduce the width $W$ of the channel in the same way that leads to a shrinkage of the constriction. The shrinkage should shift the magnetoplasmon frequency and thus change the position of the resonances on the magnetic-field axis. According to Fig. (IV.6), the positions of magnetoplasmon features do not change with $V_g$. However, taking into account the QPC's threshold voltage ($V_{th} = -2.8$ V), we can estimate that the channel shrinks by $\sim 160$ nm per 1 V of the gate polarization. Then, the position of the fundamental magnetoplasmon resonance should shift to lower fields by $\Delta B \sim 0.007$ T per 1 V of the gate polarization, an amount which is much smaller than a typical half-width (0.16 T) of the resonance peaks observed.

In addition, a THz photocurrent excited in the QPC structure was investigated as a function of magnetic field and different laser frequency. In Fig. (IV.8) (a), photocurrent spectra as a function of $B$ are shown for the laser wavelengths $\lambda_1 = 118.8 \mu m$ and $\lambda_2 = 163 \mu m$. Both lateral gates were biased with a gate voltage $V_g = 0$ V. The observed CR peaks are denoted by the solid triangles and inscriptions "CR" for each spectra. As previously, cyclotron effective mass was found to be equal to $\sim 0.103m_0$. The spectral structures at the low-$B$ side of CR maxima (positions of the peaks are marked with open triangles) arise due to excitation of the fundamental and higher order modes of magnetoplasmons. Also, in the spectrum measured at 163 $\mu m$ the SdH oscillations appear.

Fig. (IV.8) (b) shows experimentally determined plasmon dispersions for the laser wavelengths $\lambda = 118.8 \mu m$ and $\lambda = 163 \mu m$ (marked by circles and triangles, respectively). The wavevectors were normalized to the fundamental plasmon mode wavevector $k_1$. Plasmon frequency was obtained from magnetoplasmon positions in magnetic field with the aid of Eq. (IV.6).

Similarly as before, we take that the plasmon wavevector is $k_j = \pi j/W$, where $W = 2.4 \mu m$ is the width of a conduction channel and $j$ is a positive integer.

Plasma dispersion fit (with $\eta$ and $\varepsilon_{b,\infty}$ as fitting parameters) described by Eqs. (V.10) and (IV.7) and assuming the above discussed wavevector $k_j = \pi j/W$ is represented by a solid line in the inset to Fig. (IV.8). The values of fitting parameters were found to be $\eta = 0.95$ and $\varepsilon_{b,\infty} = 6.1$. This is in a good agreement with the results obtained in the case of
Figure IV.8. (a): photocurrent spectra of quantum point contact sample at laser lines 118.8 μm (2.52 THz) and 163 μm (3.11 THz). Rich spectral structures are identified as cyclotron resonance (denoted by inscription "CR" and solid triangles) accompanied by the fundamental and higher-order magnetoplasmon harmonics (marked by open triangles). In 163 μm spectrum, the Shubnikov-de Hass oscillations are seen as well. The voltage $V_g = 0$ V was provided symmetrically on a split-gate.

(b): Experimental frequencies of plasmon modes for 118.8 μm (circles) and 163 μm (triangles) fitted with a theoretical dispersion relation including plasmon-LO phonon interaction (solid curve).

Plasmons observed at the experiment of constant frequency. As it was found previously, here the plasmons are practically ungated with a weak screening most likely arising due to the presence of lateral gates. Assuming only odd plasmon modes would introduce significantly larger 2DEG screening; however, as it was mentioned already, there is no evident reason for it to occur.
V. RESONANT TERAHERTZ DETECTION IN GATED 2DEG PLASMA

Here we will present results of a THz detection experiments on a gated 2DEG in a high electron mobility Cd(Mn)Te/CdMgTe QWs in a constant magnetic field and as a function of a gate voltage. It was demonstrated in the previous chapter that, in principle, THz radiation can be used to induce collective oscillations of a two-dimensional electron plasma. This points to possible applications of CdTe-based QWs for fabrication of a FET for THz detection, governed by a plasma instability in a transistor channel [123, 124].

Experiments were performed in a quantizing magnetic field and $T = 1.8$ K. The samples of interest were gated Hall bar (HB), and a gated bar of the crystal with two-terminal configuration (P5). A detailed description of the devices can be found in Section I. We have used two laser frequencies (2.52 and 3.11 THz) to excite the photocurrent in a sample, kept in the magnetic field, such that condition $\omega_l = \omega_c$ was satisfied (9.07 and 11.46 T, respectively). Detection signal was recorded as a photocurrent in a constant magnetic field as a function of gate voltage (electron concentration), down to the threshold voltage $V_{th}$, at which no electrons under the gate remained. The spectra were taken at bias voltages $V_{ds}$ in the interval 20–100 mV, with a step of 20 mV. During the experiment, samples were connected in the usual manner of a field-effect transistor, with a drain-source voltage $V_{ds}$ supplied between drain contact and source contact connected with a ground of experimental setup, while $V_g$ was connected between the gate and source contacts. The $I$-$V$-characteristics of the devices [Figs. (V.1) (a) and (b)] show that the conduction channel is of $n$–type and the electron concentration under the gate is controlled by a $V_g$ due to a field-effect. At $V_{th}$, the channel is depleted and no current can pass through. Also, a gate leakage current was negligibly small, below 1 nA, the low-measure limit of our equipment.

A photocurrent (a magnitude of CR peak, in fact) registered as a function of $V_g$ at a constant magnetic field for the HB sample is represented by the solid curves in Figs. (V.2) (a) and (b) at laser frequencies of 2.52 THz and 3.11 THz, respectively. The Landau level filling factor $\nu$ calculated corresponding to $V_g$ is put on the secondary (upper) $x$-axis. Also, DC transport measurements (sample resistance) in two-terminal configuration conducted simultaneously to photocurrent measurements are depicted in the same figure as dashed curves. The resistance curves, recorded at the different $V_{ds}$ values, were at the same level, but for convenience they were shifted in the positive direction of $y$-axis. The bottom curve
Figure V.1. IV characteristics of HB sample (a) and P5 sample (b).

corresponds to \( V_{ds} = 20 \, \text{mV} \) and the top curve corresponds to \( V_{ds} = 100 \, \text{mV} \). We do not show the resistance at the gate voltage close threshold voltage \( V_{th} = -0.8 \, \text{V} \), where it rises to the high values very fast and photoreponse goes to zero.

From comparison of CR magnitude and DC transport dependencies on gate voltage, we conclude that these two signals are inversely proportional. The dependencies demonstrate a step-like behaviour, with approximately constant values of the photocurrent signal around integer \( \nu \), which resembles the plateaus of Hall resistance at high magnetic field. The formation of IQH states in this sample was shown by a double-peak occurrence in the photocurrent dependence on a magnetic field [see subsection III B 1 and Fig. (III.4) (d) therein].

In a quantum Hall regime, the 2D bulk is insulating, and conductance occurs only through the edge states of the sample, thus a decrease in photosignal should occur. However, we observe the cascade behaviour of the photocurrent instead of oscillations with minima at integer filling factors. Also, the step between filling factors 2 and 3 is not observed neither in resistance nor in photocurrent. It is most probably due to the fact that there is no IQH state at \( \nu = 3 \). The next IQH state should be observed at \( \nu = 4 \) but it is out of the scale of the figure. However, and slope in photocurrent is observed which starts at LL filling factor of 2.5.

Also, we have observed the increase in photocurrent signal when bias voltage \( V_{ds} \) is increased. The higher value of drain-source voltage contributes to electron heating, thus the signal increases. In addition, no change in a shape of the spectrum was observed when
Figure V.2. THz induced photocurrent registered on a Hall bar (HB) sample as a function of gate voltage $V_g$ at excitation frequencies of (a): 2.52 THz and (b): 3.11 THz in a constant magnetic field. Measurements were performed at bias voltage $V_{ds} = 20-100$ mV at every 20 mV. Photocurrent was normalized to laser power and is given in arbitrary units. Shown with dashed curves is the DC resistance of the sample, measured simultaneously with the photocurrent.
drain-source bias was changed.

The photocurrent spectra (solid curves) and DC magnetoresistance (broken curves) for the sample P5 are shown in Figs. (V.3) (a) and (b). The measurements were done for the same frequencies and values of $V_{ds}$ as in the case of Hall bar. The CR magnitude is inversely proportional to the resistance – the same was observed on Hall bar. There no plateaus observed in resistance. At $V_g = 0$ V for the frequency of 3.11 THz, the condition of $\omega_l = \omega_c$ (at $B = 11.46$ T) is satisfied when $\nu = 0.94$, therefore no integer QH states are observed when approaching $V_{th}$. However, going to the positive gate voltages, the filling factor equal to 1 is reached, what is seen in the resistance and in photocurrent as a flat part of the curves in the range of positive gate voltages. At frequency 2.52 THz the cyclotron resonance for zero-gate bias occurs at $\nu = 1.19$, and the condition $\nu = 1.00$ is satisfied to rough estimation at $V_g \approx -0.2$. Here no step-like behaviour was observed in photocurrent and magnetoresistance. However, a region with a very slight inclination (especially for the lower values of $V_{ds}$) is observed in photocurrent, at gate voltage ranging from $-0.2$ to $-0.4$ V, which might come from the QHE. On the other hand, resistance dependencies are not totally flat in this range of $V_g$ as well. The flanks of the structure in photocurrent spectra outside of the gate voltages mentioned above, follow the inverse resistance and decrease to the level of noise.

However, more detailed experiments are needed in order to clarify the nature of the observed photocurrent behaviour.

Also, we have tried THz detection on our samples in a cryogenic temperature and zero magnetic field, but the absence of photosignal suggests no contribution from the mechanism of plasma instability.
Figure V.3. THz induced photocurrent registered on sample P5 as a function of gate voltage $V_g$ at excitation frequencies of (a): 2.52 THz and (b): 3.11 THz in a constant magnetic field. Measurements were performed at bias voltage $V_{ds} = 20–100$ mV at every 20 mV. Photocurrent was normalized to laser power and is given in arbitrary units. Shown with dashed curves is the DC resistance of the sample, measured simultaneously with the photocurrent.
SUMMARY AND CONCLUSIONS

To sum up, we have presented the experimental investigation of a two-dimensional electron gas (2DEG) based on high electron mobility CdTe/CdMgTe single quantum wells with $n$-type modulation doping at one barrier. Samples of various design and shape were investigated at low temperatures by the means of magnetotransport in the dark and terahertz (THz) magnetospectroscopy in photocurrent and transmission mode. In order to explain obtained results within suitable theoretical models, determination of basic parameters such as electron concentration and electron effective mass was required.

To determine a 2D electron concentration we have measured magnetotransport in a two contact configuration at a constant bias voltage and observed oscillations in resistance arising due to a Shubnikov-de Haas effect. In the gated samples, it was shown that the concentration of the electrons changes with the gate polarization. Also, at the temperature of 1.8 K we have observed a fractional quantum Hall effect (FQHE) state at a Landau level (LL) filling factor $\nu = 3/2$. Yet, the states of integer quantum Hall effect (IQHE) could not be identified clearly in the magnetotransport experiments due to the two-terminal geometry of an experiment.

An effective electron mass was determined from THz induced photocurrent and THz transmission experiments in the magnetic fields. In a photocurrent measurements, besides the cyclotron resonance (CR) transition, oscillations periodic in the inverse magnetic field were detected and were described as coming from a non-resonant 2DEG heating by THz radiation. These oscillations were phase shifted by a 90 deg when compared with resistance (SdH) oscillations in the magnetotransport. At LL filling factors $\nu = 1, 2, 4$ a splitting of the peaks coming from a non-resonant heating was observed and was qualitatively explained as a manifestation of breaking of the IQH states in photocurrent.

THz transmission experiments revealed the electron cyclotron effective mass increase in a strong magnetic fields due to a resonant polaron effect which occurs in polar materials when the radiation frequency approaches the frequency of longitudinal optical phonon. We have also found a local increase in electron cyclotron effective mass at LL filling factor $\nu = 2$ which helped us to define the electron sheet concentration and showed a possibility to determine electron concentration from magnetotransmission experiments.

THz transmission in the samples equipped with grating couplers showed a CR accompanied with magnetoplasmon resonances. A mixed nature of these waves has been revealed.
by data analysis which shows that they are neither of a purely gated nor purely ungated type with a ratio of 1:0.8, close to the geometrical aspect ratio (1:1) of the coupler. In the quantum point contact structure, a series of practically ungated magnetoplasmon modes has been observed in the photocurrent. In each case, the dispersion relation of plasma oscillations was successfully described within a local approximation of the magnetoconductivity tensor of a two-dimensional electron gas, in spite of the fact that the value of the non-local parameter $X$ was as high as 0.25 for the magnetoplasmon modes of the highest order. Yet, this result remains in full agreement with non-local calculations which indicate that for the structures under study, the validity of a local approximation should hold up to $X = 0.32$. Also, an increase of the cyclotron effective mass due to a resonant polaron effect as well as an influence to a 2DEG screening coming from the plasmon-LO phonon interaction must be taken into account to describe magnetoplasmon dispersion relations properly.

To the best of our knowledge we have observed magnetoplasmon excitations in CdTe/CdMgTe quantum wells directly in a far-infrared magnetospectroscopy experiment for the first time.

Though our experimental work on the far-infrared magnetospectroscopy of a high electron mobility CdTe/CdMgTe reveals a manifestation of some fundamental physical phenomena, it is not completed so far. There are at least a few points of attention which could be considered in the future studies.

Due to the lack of reliable ohmic contact preparation method, the Hall transport experiments were not performed in our work. At low magnetic fields they could give an information about electron mobility and concentration in the samples. These two quantities could be compared with the values determined from SdH effect and cyclotron resonance peak width providing the additional information about the sample quality. In the high magnetic fields, Hall experiments could be used to observe IQH states and to check their behaviour in the condition of a 2DEG heating induced by a THz radiation. Also, the observed FQH state at $\nu = 3/2$ would be characterized more accurately.

Unfortunately, grid-gated samples characterized in the magnetotransmission experiments did not have functioning contacts to pass the current. This prevented us from the photocurrent experiments, which could allow to check if plasmons can be observed in photocurrent and if yes, to compare their dispersions obtained in photocurrent and in transmission.

Also, there was a set of samples with grid-couplers based on a Cd(Mn)Te/CdMgTe QWs prepared, which was not measured in the THz magnetotransmission experiments. In this
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LIST OF SYMBOLS

\( \alpha \) Fröhlich constant  
\( A \) vectorial potential of magnetic field  
\( B \) magnetic field  
\( B^* \) effective magnetic field  
\( B_0 \) magnetic flux density  
\( B_\perp \) magnetic field perpendicular to quantum well plane  
\( \chi \) polarizability of 2DEG  
\( C_e \) specific heat of free electrons  
\( C_l \) specific heat of crystal lattice  
\( \Delta \varepsilon_Z \) Landau level energy shift due to Zeeman effect  
\( D(\epsilon) \) density of states  
\( \varepsilon \) dielectric function  
\( \bar{\varepsilon}_g \) dielectric function of gated plasmons  
\( \bar{\varepsilon}_{ug} \) dielectric function of ungated plasmons  
\( E \) electric field  
\( \epsilon_\parallel \) energy parallel to quantum well plane  
\( \epsilon_\perp \) energy perpendicular to quantum well plane  
\( \epsilon_n \) energy of free electron system at \( B = 0 \) T  
\( \varepsilon \) static dielectric constant  
\( \varepsilon_\infty \) high frequency dielectric constant  
\( \varepsilon_b \) static dielectric constant of QW barrier  
\( \varepsilon_a \) static dielectric constant of QW  
\( e \) elementary charge \( e = 1.6 \times 10^{-19} \) C  
\( E_H \) Hall electric field  
\( E_x \) x-component of electric field  
\( \Gamma \) Landau level broadening  
\( g \) electron Landé factor  
\( \hbar \) reduced Planck’s constant \( \hbar = \hbar/2\pi = 1.055 \times 10^{-34} \) Js  
\( h \) Planck’s constant \( h = 6.626 \times 10^{-34} \) Js  
\( I_{gs} \) gate leakage current  
\( I_p \) photocurrent  
\( I_{th} \) drain-source current  
\( j \) current density  
\( j_H \) Hall current density  
\( k_\perp \) electron wavevector perpendicular to quantum well plane  
\( k_{p,j} \) plasmon wavevector of \( j \)-th harmonic  
\( \Lambda \) grid coupler period  
\( l_B \) magnetic length  
\( \mu \) electron mobility  
\( \mu_B \) Bohr’s magneton \( \mu_B = e\hbar/2m_0 = 9.274 \times 10^{-24} \) K/T  
\( m_b \) electron band mass  
\( m_c^* \) cyclotron effective mass  
\( m_e^* \) electron effective mass  
\( m_p^* \) polaron mass  
\( \nu \) Landau level filling factor
\( n_s \) 2D electron concentration
\( \Phi \) electrostatic potential
\( \Phi_0 \) magnetic unit flux
\( P \) polarization of 2DEG
\( \rho_0 \) resistivity at \( B = 0 \) T
\( \rho_{\text{ind}} \) electromagnetic wave induced space charge
\( R_0 \) sample resistance at zero magnetic field
\( R_a \) amplitude of Shubnikov-de Haas peak
\( R_H \) Hall resistance
\( s \) electron spin operator
\( \sigma_0 \) conductivity at \( B = 0 \) T
\( \tau_{\text{tr}} \) transport scattering time
\( \tau_q \) quantum scattering time
\( T_{\text{crit}} \) critical temperature
\( T_e \) electron temperature
\( T_l \) lattice temperature
\( v_d \) electron drift velocity
\( V(z) \) vectorial potential of magnetic field
\( V_{\text{ds}} \) drain-source voltage
\( v_F \) Fermi velocity
\( V_g \) gate voltage
\( V_{g_{\text{ac}}} \) gate voltage with small-amplitude modulation
\( V_{g_{\text{dc}}} \) constant gate voltage
\( V_{\text{th}} \) threshold voltage
\( \omega_c \) electron cyclotron frequency
\( \omega_{\text{LO}} \) LO phonon frequency
\( \omega_{\text{mp}} \) magnetoplasmon frequency
\( \omega_p \) plasmon frequency
\( \omega_{\text{TO}} \) TO phonon frequency
\( X \) non-local parameter
\( x, y \) coordinates of quantum well plane
\( z \) growth direction of a quantum well


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