Summary

The main subject of the research presented in the dissertation is the Coxeter spectral classification of finite connected partially ordered sets (posets) \( I \), encoded in the form of incidence matrices \( C_I \in \mathbb{M}_{|I|}(\mathbb{Z}) \), by means of symbolic and combinatorial algorithms. We consider posets \( I \) with symmetric Gram matrix \( G_I := \frac{1}{2}(C_I + C_I^t) \in \mathbb{M}_{|I|}(\mathbb{Z}) \) positive definite (positive posets) or positive semidefinite of rank \( |I| - r \) (nonnegative posets of corank \( r \)) and we classify them up to the \( \mathbb{Z} \)-congruence of incidence matrices (the relation \( \approx \mathbb{Z} \)), and \( \mathbb{Z} \)-congruence of symmetric Gram matrices (the relation \( \sim \mathbb{Z} \)).

The main problem considered in the dissertation is the problem of the existence of invariants that define a connected nonnegative poset \( I \) uniquely, up to the relation \( \approx \mathbb{Z} \). We show that in the case of positive posets with exactly one maximal element (i.e. one-peak posets) or at most \( |I| \leq 14 \) elements, such invariants are: the complex spectrum \( \text{specc}_I \subseteq \mathbb{C} \) of the Coxeter matrix \( \text{Cox}_I := -C_I \cdot (C_I^t)^{-1} \in \mathbb{M}_{|I|}(\mathbb{Z}) \) and the Dynkin type \( \text{Dyn}_I \in \{A_{|I|}, D_{|I|}, E_6, E_7, E_8\} \), uniquely associated to \( I \). Furthermore, we show that in the case of considered positive posets the relations \( \sim \mathbb{Z} \) and \( \approx \mathbb{Z} \) coincide. Next we present analogous results for the case of connected nonnegative posets of corank \( r \in \{1, 2\} \). In particular, we show that the pair \( (\text{specc}_I, \text{Dyn}_I) \) is a good invariant in case of such posets.

The second important problem considered in the dissertation is the construction of algorithms, that find an \( \mathbb{Z} \)-invertible matrix \( B \in \mathbb{M}_n(\mathbb{Z}) \) that defines the \( I \approx \mathbb{Z} J \) relation between connected nonnegative posets \( I \) and \( J \) of \( n \) elements, i.e. satisfy the equality \( C_I = B^t \cdot C_J \cdot B \). We present two algorithms that solve this problem: first one is an exhaustive search algorithm, that guarantees to find a solution in a case of positive posets and the second one is the heuristic algorithm for a more general class of nonnegative posets.

The main results of the dissertation are: (a) algorithms that construct matrices defining the relation \( \approx \mathbb{Z} \); (b) the proof that, for broad classes of finite connected partially ordered sets, the following relations

\[
I \approx \mathbb{Z} J \iff I \sim \mathbb{Z} J \iff \text{specc}_I = \text{specc}_J \iff \text{Dyn}_I = \text{Dyn}_J
\]

hold; (c) a complete Coxeter spectral classification, up to the relations \( \sim \mathbb{Z} \) and \( \approx \mathbb{Z} \), of broad classes of finite partially ordered sets; (d) a complete Coxeter spectral classification, up to isomorphism, of one-peak posets that are positive or nonnegative of corank 1.