Perturbative Corrections to the Inclusive Radiative $B$ Meson Decay

PhD thesis

Abstract

The main subject of my Ph.D. thesis is the calculation of the unknown corrections to the radiative $b$ quark decay $b \to X^p_\gamma$ which at the hadronic level corresponds to the radiative $B$ meson decay $\bar{B} \to X_s \gamma$. Perturbatively defined decay rate $\Gamma(b^- \to X^p_\gamma)$ approximately equals to the $\bar{B} \to X_s \gamma$ decay rate.

Precise knowledge of the decay $\bar{B} \to X_s \gamma$ within the Standard Model provides powerful constraints on the theories beyond Standard Model. Two Higgs Doublet Models, supersymmetric theories or models with extra spacetime dimensions are the most significant examples.

The Ph.D. thesis includes two calculations. First, we found tree-level contributions from CKM-suppressed $b \to u\bar{u}s\gamma$ transitions together with similar ones from the QCD penguin operators. Weak radiative decay $\bar{B} \to X_s \gamma$ is known to be a loop-generated process. However, it does receive the aforementioned tree-level contributions. For a low value of the photon energy cutoff $E_0 \sim m_b/20$ that has often been used in the literature, they can enhance the inclusive branching ratio by more than 10%. For $E_0 = 1.6$ GeV or higher, the effect does not exceed 0.4%, which is due to phase-space suppression and smallness of the relevant Wilson coefficients. This tiny numerical value does not exceed previous rough estimations and justifies why these contributions have been neglected up till now. The perturbative results contain collinear logarithms that depend on the light quark masses $m_q$ ($q = u, d, s$). We have allowed $m_b/m_q$ to vary from 10 to 50, which corresponds to values of $m_q$ that are typical for the constituent quark masses.

Second part of the Ph.D. thesis is devoted to calculation of the perturbative $O(\alpha_s^2)$ corrections to the branching ration $\text{BR}(B \to X_s \gamma)$ in the Brodsky-Lepage-Mackenzie approximation. They receive contributions from two-, three- and four-body final states. While all the two-body results are well established by now, the other ones have remained incomplete for several years. We have calculated the last contribution that has been missing.
to date, namely the one originating from interference of the current-current and gluonic
dipole operators ($K_{18}^{(2)β0}$ and $K_{28}^{(2)β0}$). Moreover, we confirmed all the previously known
results for BLM corrections to the photon energy spectrum that involve the current-current
operators (e.g., $K_{22}^{(2)β0}$ and $K_{27}^{(2)β0}$). Finally, we also confirm the findings of Ferroglia and
Haisch on self-interference of the gluonic dipole operator ($K_{88}^{(2)β0}$) which was published in
the same time.

Most of the found corrections do not exceed ±1% of the branching ratio central value.
The only exception is $K_{22}^{(2)β0}$ correction which affects the branching ratio by +1.9%, which
still remains within the assumed ±3% uncertainty for all such effects. In spite of the small-
ness of the found corrections, their knowledge is crucial for reducing the theory uncertainty
in the close future.

The thesis is supplemented with four appendices. Appendix A includes description of the
integration over four-particle phase space without neglecting light quark masses. Particular
attention is paid to the extraction of the collinear logarithms with the possibility of staying
differential in the photon energy. Appendix B is devoted to the derivation of the splitting
functions in the light-cone axial gauge. Splitting functions describe photon behaviour at
small angles relative to the emitter. They are commonly used to extract collinear logarithms
in processes with radiation. In the considered calculation however, they are used in order
to compare the results received in dimensional regularisation with those where the collinear
emission is regulated by the light quark masses. Such a comparison is important to perform
the crosscheck of the results. Appendix C includes description of Smith-Voloshin method
which was used in the calculation of the NNLO corrections in the BLM approximation.
Appendix D contains analytic functions necessary in the evaluation of the $K_{22}^{(2)β0}$ correction.